# CHARACTERISTICS OF CIRCULAR-CRESTED WEIR

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**ABSTRACT:** The circular-crested weir is used for flow measurement and has wide applications in hydraulic engineering, where it serves as an overflow structure. It can be used to control the water level in farm ponds and reservoirs. The discharge coefficient of such a weir is obtained experimentally as a function of the dimensionless total head of the approaching flow. It is shown that changing the upstream slope does not alter the weir-discharge coefficient. However, increasing the downstream weir slope appears to increase the discharge coefficient. The weir crest pressure is determined as a function of the dimensionless head and its variation is related to the weir-discharge coefficient. The experimental data indicate that the circular-crested weir behaves like a sharp-crested weir, when the dimensionless total head is extremely large.

## INTRODUCTION

The circular-crested weir is used for flow measurement and has wide applications in hydraulic engineering, where it serves as an overflow structure. It can be used to control the water level in farm ponds and reservoirs. The simplest form of such a weir consists of a circular crest of radius $R$ set tangentially to an upstream face and perpendicular to the direction of flow (Fig. 1). The weir is commonly constructed with a downstream face having a slope $\beta$ of 45° and has a high discharge coefficient $C_D$. Defining $q$ as the discharge passing over a unit length of the weir crest, $C_D$ can be related to $q$ as follows:

$$q = \frac{2}{3} C_D \sqrt{2gH_1^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

Here, $g$ = acceleration due to gravity; and $H_1$ = total head of the approaching flow reckoned above the weir crest lelvel. From direct measurement of the discharge passing over the weir model (Fig. 1), $C_D$ can be evaluated using (1). According to Bos (1978), the crest pressure $(P/\gamma)_{cr}$ also denotes the minimum pressure $(P/\gamma)_{min}$ on the crest surface. Setting $Y_2 \approx 0.7\ H_1$, and using the data of Escande and Sananes (1959), Bos (1978) related $(P/\gamma H_1)_{cr}$ with $H_1/R$. When the downstream slope of the weir $\beta$ = 45° and $H_1/R < 10$, this relation can be rewritten as

$$\left(\frac{P}{\gamma H_1}\right)_{cr} \approx 0.71 - 0.44 \left(\frac{H_1}{R}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

For flow over circular-crested weirs for which $H_1/R \leq 1.0$, Matthew (1963) outlined a simple theory that clearly explains the influence of surface tension, viscosity, and the radius of curvature of the streamline on $C_D$. Cassidy (1965)

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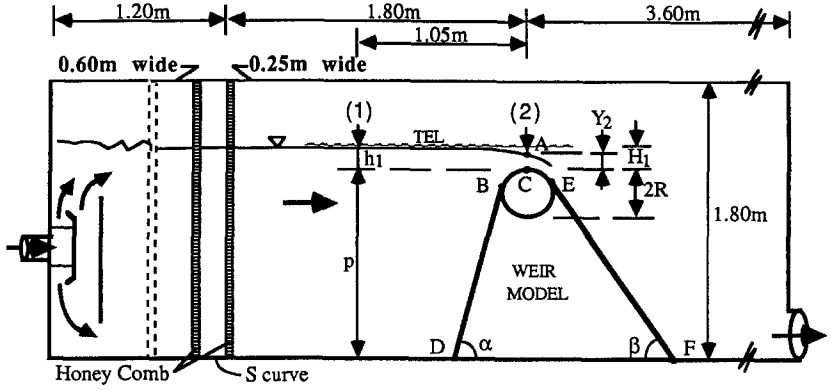


**FIG. 1. Experimental Setup: Elevation (Not to scale)**

presented ideal fluid-flow models of circular-crested weirs and standard spillways. Seshadri (1981) studied the characteristics of circular-crested weirs with low weir heights. Bos (1978) analysed existing data in detail to present a relationship between $C_D$ and $H_1/R$ for circular-crested weirs for which $\beta = 45°$. He showed that $C_D$ increases from 0.64 at $H_1/R = 0.05$ to 1.48 at $H_1/R = 5.50$, and that $C_D$ remains unchanged for $5.50 < H_1/R \leq 9.50$. An interesting study related to the gated and ungated standard spillways was reported by Sinniger and Hager (1985).

The present experimental study deals with the discharge characteristics of circular-crested weirs over a wide range of $H_1/R$ ($0.45 \leq H_1/R \leq 18.5$). As another part of the study, the dependence of $C_D$ on the parameter $H_1/R$ and the upstream and downstream slopes $\alpha$ and $\beta$ (Fig. 1) is determined. The weir crest pressure is determined as a function of $H_1/R$, and its influence on $C_D$ is discussed. The present data are also compared with earlier results, which cover the lower range of $H_1/R$. At very large values of $H_1/R$, the discharge coefficient of circular-crested weirs are shown to approach the discharge coefficient of sharp-crested weirs with the same side slopes.

## THEORETICAL CONSIDERATIONS

The following assumptions are made in the development of the model for the circular-crested weir (Fig. 1):

1. The flow upstream of the weir is steady, two-dimensional and subcritical.
2. The dimensionless weir height $p/H_1$ is large, and its effect on the weir discharge is negligible ($p/H_1 \geq 3$).

Since $p/H_1 > 3$ in the present tests, the effect of the velocity head in the approach flow relative to the flow depth $h_1$ above the crest (Fig. 1) is negligible ($h_1 \approx H_1$). Further, the effects of surface tension and viscosity on $C_d$ were eliminated to a large extent by keeping $h_1$ in the range 0.05 m $\leq h_1 \leq$ 0.18 m. Hence, $C_D$ can be expressed as

$$C_D = \frac{q}{\frac{2}{3}\sqrt{2gH_1^3}} = \pi\left(\frac{H_1}{R}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Here, $\pi$ ( ) denotes a function of $H_1/R$. Similarly, the normalized pressure head at the crest $(P/\gamma H_1)_{cr}$ can be expressed as

$$\left(\frac{P}{\gamma H_1}\right)_{cr} = \Phi\left(\frac{H_1}{R}\right) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

Eqs. 3 and 4 represent the weir flow characteristics and $\Phi$ ( ) and $\pi$ ( ) denote different functions of $H_1/R$.

## EXPERIMENTAL SETUP AND PROCEDURE

Machined plexiglass weir models were set in a smooth stainless steel flume (Fig. 1). The radii of the weir models were 0.0095 m, 0.0254 m, 0.0381 m, 0.0754 m, 0.1008 m, and 0.1516 m, and were true to the nearest $5 \times 10^{-5}$ m. The test section was 0.254 m wide, 1.80 m high, and 2.50 m long. The side walls were equipped with transparent windows for flow visualization of separating flows, based on dye injection. Sufficient stilling arrangements were provided to obtain a smooth flow approaching the crest. To study the effects of weir slopes on $C_D$, tests were performed on weir models with combinations of upstream slopes $\alpha$ and downstream slopes $\beta$. The slopes were $\alpha = 90°$, 75°, and 60° in combination with downstream slopes $\beta =$ 75°, 60°, and 45°. Tests were also conducted on sharp-crested weir models with sharp machined edge at the apex ($1.5 \times 10^{-4}$ m imperfection). These denoted the limiting case of a circular-crested weirs ($R/H_1 \to 0$). All models were equipped with sufficient pressure taps of diameter $0.5 \times 10^{-3}$ m to record the pressure distribution along the model centerline. On the vertical weir face, pressure taps were spaced at every 0.05 m along the centerline. For models with $R \geq 0.0254$ m, taps were spaced at 5° intervals on the weir crest. The pressure head was recorded to the nearest $0.5 \times 10^{-3}$ m. The water-surface profiles over the weir crest and the flow depth upstream of the weir were measured by means of point gages (Fig. 1) to the nearest $0.3 \times 10^{-3}$ m. The flow rate was determined by measuring the flow depth over a standard 60° V-notch. The maximum error in the determination of the flow rate was estimated to be 3%.

## RESULTS

### Variation of $C_D$ with $H_1/R$

Fig. 2 shows the experimentally determined relationship between $C_D$ and $H_1/R$ for circular-crested weirs for which $\alpha = 90°$ and $\beta = 45°$. For larger $H_1/R$, Bos (1978) used the data of Escande (1959) whose weir crest had a slot and showed that $C_D$ attains an upper limiting value of 0.86 near $H_1/R = 5$ and remains unchanged for larger $H_1/R$ (curve A, Fig. 2). The present data shows that $C_D$ attains a maximum value of 0.87 as $H_1/R$ is increased to 5.5 and then decreases with a further increase in $H_1/R$ (curve B, Fig. 2). This discrepancy may be traced in part to the effect of the slot on $C_D$ in the tests of Escande (1959). Fig. 3 denotes the expanded scale version of all the earlier data used by Bos (1978) to form the $C_D - H_1/R$ relation. It indicates that, even in the earlier studies, $C_D$ increases gradually with an

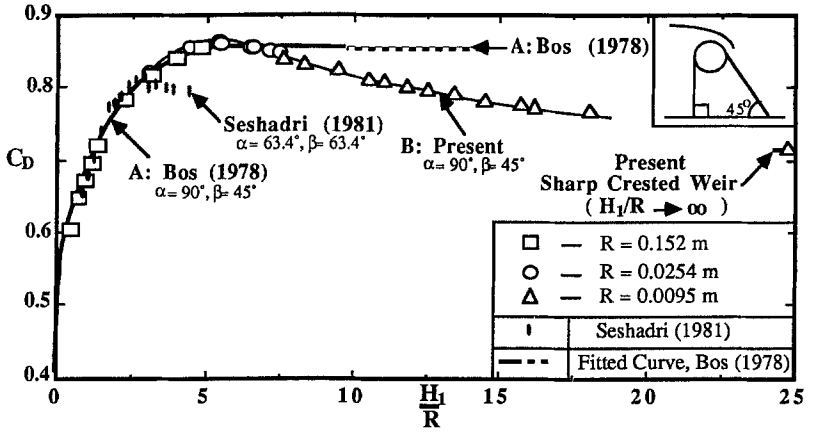


**FIG. 2. Variation of $C_D$ with $H_1/R$ $0 < H_1/R \leq 25$**

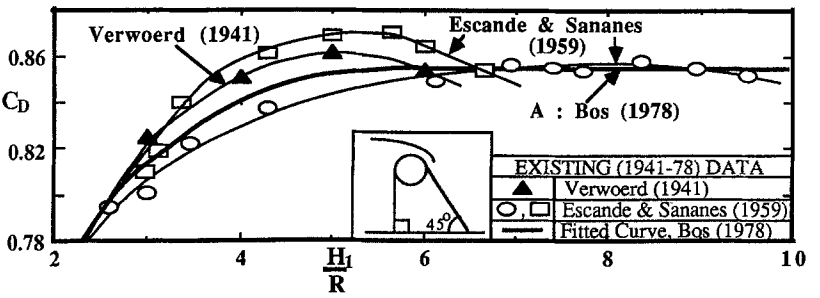


**FIG. 3. Variation of $C_D$ with $H_1/R$ $2 \leq H_1/R \leq 10$**

increase in $H_1/R$, attains a maximum value, and later decreases with a further increase in $H_1/R$. Fig. 2 also includes the results of more recent studies (Seshadri 1981), which confirm qualitatively the fact that $C_D$ attains a maximum as $H_1/R$ increases and later decreases with a further increase in $H_1/R$. For a weir with fixed $R$, both the subatmospheric pressure at the crest and $C_D$ increase when $H_1/R$ is increased, since the flow profile over the crest becomes steeper and more curved. However, when $H_1/R$ is larger than five, the flow separates in the region BC (Fig. 1), and no further increase in $C_D$ occurs. Further, at very large $H_1/R$, the value of $C_D$ should drop and reach the value of $C_D$ for sharp-crested weirs. The present experimental values of $C_D$ for sharp-crested weir ($\alpha = 90°$, $\beta = 45°$) are also included in Fig. 2 to indicate that the circular-crested weirs behave like a sharp-crested weir at large $H_1/R$ values ($H_1/R \rightarrow \infty$).

## Crest Pressure

For circular-crested weirs with varying degrees of side slopes and $R \geq 0.0254$ m, the normalized crest pressure head $(P/\gamma H_1)_{cr}$ denoting the pressure at C (the highest point of the crest as shown in Fig. 1) is shown as a function of $H_1/R$ in Fig. 4. It also shows that the dependence of $(P/\gamma H_1)_{cr}$ with $H_1/R$ agree qualitatively with the approximate relation suggested by Bos (1978). He assumed $(P/\gamma H_1)_{cr} = (P/\gamma H_1)_{min}$. Since only the downstream

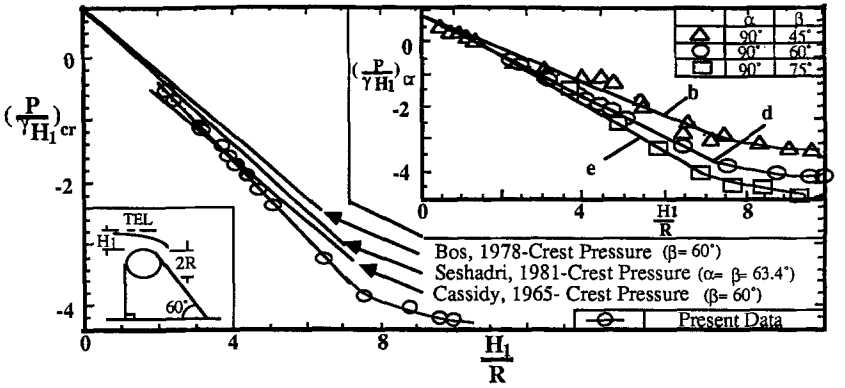


**FIG. 4. Variation of $(P/\gamma H_1)_{cr}$ with $H_1/R$, Insert Shows Effect of β on $(P/\gamma H_1)_{cr}$**

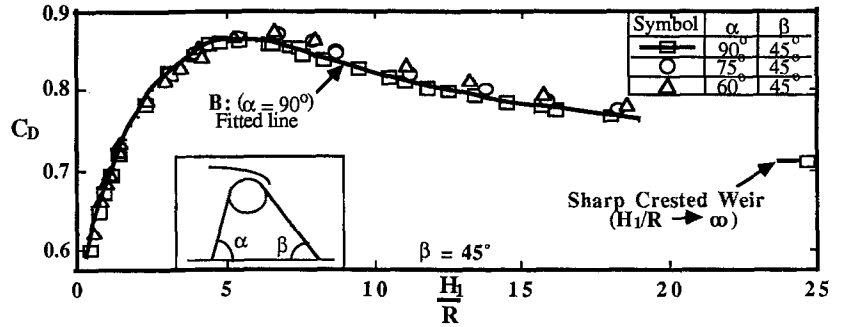


**FIG. 5. Effect of Upstream Slope α on $C_D$ − (0 < $H_1/R \leq 25$)**

slope is influential in affecting $C_D$ and $(P/\gamma H_1)_{cr}$, the data of Seshadri (1981) for symmetric weirs ($\alpha = \beta = 63.4°$) are also included in Fig. 4 for comparison. His data compare well with the relation proposed by Bos (1978) and the present data ($\alpha = 90°$ and $\beta = 60°$). The present crest pressure data (Fig. 4) also agree well with the results of the ideal flow model of Cassidy (1965). The decrease in $(P/\gamma H_1)_{cr}$ due to an increase in $H_1/R$ is arrested near $H_1/R = 8$. This corresponds to the value of $H_1/R$ at which $C_D$ is a maximum.

### Effect of Upstream Weir Slope α on $C_D$

A gentle upstream slope contributes to the streamlining of the weir shape and tends to increase $C_D$. However, it adds to the upstream area over which flow occurs. This will tend to decrease $C_D$. The resulting effect on $C_D$ is not significant in the range of tests covered (Fig. 5). Fig. 5 shows the variation of $C_D$ with $H_1/R$ for weirs with fixed downstream slope β and varying upstream slope α. The data indicate that the variation of α does not significantly change $C_D$ at fixed $H_1/R$ and fixed β. For instance, the deviation in $C_D$ is at most 2% (Fig. 5) when α is changed from 90° to 60° in the entire range of tests ($0.45 \leq H_1/R \leq 18.5$) for the weir with a fixed downstream slope ($\beta = 45°$). According to the test results, the changes in the upstream slope α did not significantly alter the value of $C_D$ when the downstream

slope β was fixed at 60° and 75°. It may be added that these results compare favorably with the nearly constant upstream slope correction factor suggested by the Waterways Experiment Station (WES) for $C_D$ of standard spillways (Bos 1978). For these spillways, α ranged from 45° to 90°, and $p/H_1$ was greater than 1.5.

**Effect of Downstream Weir Slope β on $C_D$**

For $0 < H_1/R \leq 3.5$ and $\alpha = 90°$, curves E, D, and B of Fig. 6 indicate that $C_D$ does not vary with the downstream slope ($45° < \beta < 75°$). However, for $H_1/R > 3.5$ and a fixed upstream slope $\alpha = 90°$, a steeper downstream slope increases $C_D$. This peak value of $C_D$ occurs at a higher $H_1/R$ value, when β is increased gradually (Fig. 6). Escande and Sananes (1959) also observed that a steeper downstream slope improves the weir performance. It should, however, be noted that the flow at very high $H_1/R$ is less stable when β values are very large ($\beta > 60°$). The variation of $C_D$ and $H_1/R$ given by Cassidy (1965) for the ideal flow model of circular-crested weir is also shown in Fig. 6. The deviation between the results based on ideal flow and the present results can be traced in part to the presence of both the wall and crest boundary layers and the existence of flow separation in real flows. The data for two sharp-crested weirs that indicate the limiting case of the circular-crested weir ($H_1/R \rightarrow \infty$) are also included in Fig. 6 to indicate the value of $C_D$ for large values of $H_1/R$. For $H_1/R > 3$, flow separation occurs ahead of C (Fig. 1), and the separating streamline (before reattaching downstream of C) will have a slightly steeper curvature when β is larger. Hence, at fixed $H_1/R$ values, slightly lower $(P/\gamma H_1)_{cr}$ and higher $C_D$ values are registered at larger β values.

**Correlation between $C_D$ and $(P/\gamma H_1)_{cr}$**

The boundary layer thickness δ on the crest is thin ($\delta << H_1$), and the flow outside the boundary layer is irrotational. Hence, the crest pressures $(P/\gamma H_1)_{cr}$ are expected to be related directly with the maximum theoretical velocity $U_1 \approx \sqrt{2g(H_1 - (P/\gamma)_{cr})}$ at the edge of the boundary layer where the pressure is close to the crest pressure. Lower crest pressures are associated with higher values of $U_1$, and thus improve the weir performance in terms of $C_D$. The insert of Fig. 4 shows the variation of the crest pressure

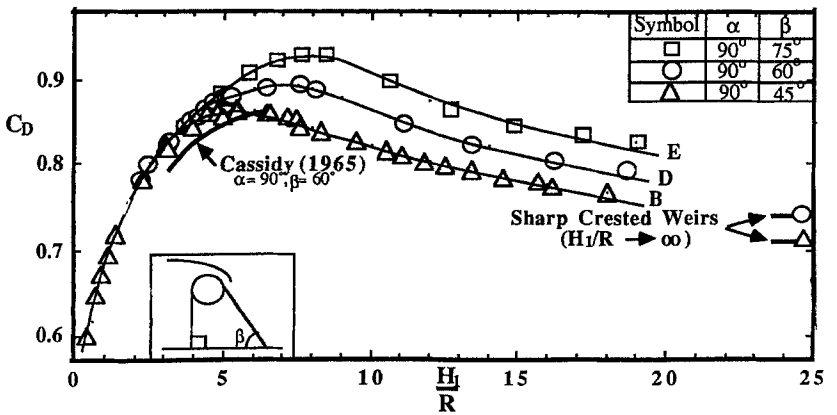


**FIG. 6. Effect of Downstream Slope β on $C_D$ – ($0 < H_1/R \leq 25$)**

for circular-crested weirs with various slopes. Here, the curves b, d, and e should be viewed with the corresponding curves, B, D, and E of Fig. 6. Fig. 6 and the insert in Fig. 4 show that up to $H_1/R$ close to 2.5, $(P/\gamma H_1)_{cr}$ and $C_D$ are essentially the same for all the models tested. Further, they also indicate that for $H_1/R > 2.5$, variations in $C_D$ values are correlated with the crest suction pressures attained at various $H_1/R$ and $\beta$ values.

## CONCLUSIONS

For the commonly used circular-crested weir with $\alpha = 90°$ and $\beta = 45°$, the variation of $C_D$ with $H_1/R$ in the lower range $0 < H_1/R \leq 5.5$ generally agrees very well with the results of previous studies. However, for $H_1/R > 5.5$, $C_D$ decreases, and for very large values of $H_1/R$, the $C_D$ value approaches the $C_D$ value for sharp-crested weirs with the same side slopes. A somewhat similar trend in the $C_D - H_1/R$ relationship is noticed for weirs with other upstream and downstream slopes.

For a fixed downstream slope, the effect of upstream weir slope on $C_D$ is marginal. For a fixed upstream weir slope, for $3 < H_1/R$, $C_D$ increases when the downstream slope is increased.

The variations in $C_D$ are directly correlated with the crest suction pressures $(P/\gamma H_1)_{cr}$ for all the weir types tested.

## APPENDIX I. REFERENCES


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## APPENDIX II. NOTATION

*The following symbols are used in this paper:*

$C_D$ = coefficient of discharge;
$g$ = acceleration due to gravity;
$H$ = total head reckoned above crest level;
$h$ = piezometric head, flow depth above crest level;
$P$ = pressure;
$p$ = weir height;
$q$ = discharge per unit width;
$R$ = radius of circular crest;
TEL = total energy line;

$U_1$ = maximum velocity at crest, ideal flow ($U_1 = \sqrt{2g(H_1 - (P/\gamma)_{cr})}$);
$x$ = $x$-direction, horizontal distance;
$Y$ = total flow depth;
$y$ = $y$-direction, vertical distance, depth from bed;
$\alpha$ = upstream slope angle;
$\beta$ = downstream slope angle;
$\gamma$ = specific weight of water;
$\delta$ = boundary layer thickness; and
$\pi$, $\Phi(\ )$ = function of.

**Subscripts**

1 = approach section 1, index;
2 = downstream section 2, index;
cr = at weir crest; and
min = at minimum value.