UPPER BOUNDARY CONDITIONS FOR OVERLAND FLOW

By J. L. M. P. de Lima[1] and P. J. J. F. Torfs[2]

INTRODUCTION

The solutions to equations of continuity and motion for overland flow, whether analytical or numerical, require the specification of initial and boundary conditions. The upstream boundary condition most frequently used in overland-flow modeling is $h(0,t) = 0$ for $t \geq 0$ [Fig. 1(*a*)], where $h(x,t)$ is the flow depth as a function of position x and time t. On watersheds with steep slopes, this condition is valid, but for moderate-to-gentle slopes, its validity is questionable (Singh 1978). Mathematical tractability is perhaps one reason for its use.

The hydraulics of overland flow on smooth surfaces and the effect of simulated rainfall have been treated extensively (Yu and McNown 1964; Yoon and Wenzel 1971; Savat 1977). Robertson et al. (1966), Kilinc and Richardson (1973), and many other researchers assumed an overland-flow profile like the one presented in Fig. 1(*a*). Shen and Li (1973) did experiments on overland flow over smooth surfaces caused by a constant base flow and constant rainfall rate. They studied the effect of different boundary conditions. Fig. 1(*b*) shows one of these, for a fully supercritical regime. Lima (1989a, 1989b) studied the effect of oblique rainfall on the overland-flow process and verified that the impact of inclined raindrops and the shear stress caused by wind (blowing upslope) at the water surface can create a discharge at the upstream boundary of a plane (depending strongly on slope, rainfall intensity, and wind speed). This implies the existence of a nonzero water depth at $x = 0$ [Fig. 1(*c*)].

To learn more about the influence of upstream boundary conditions on the hydraulics of overland flow, a laboratory experiment was undertaken. Case d (Fig. 1) was chosen for the laboratory setup. It tries to represent saturation overland flow (Dunne and Leopold 1978), where excess rainfall only begins at a certain distance from the top of the slope. This occurs frequently when soils become saturated at the surface because of rising water tables. The increase of water depth observed in this situation could play a role in water erosion (splash and sheet erosion) studies.

For case d, the resistance coefficients for steady uniform turbulent flow are not applicable (flow rate may be zero at $x = 0$, but there is also a nonzero water depth), nor is the initial flow unidirectional, as water will also flow upslope at $x = 0$ (as observed with dye tracing). Moreover, the surface-

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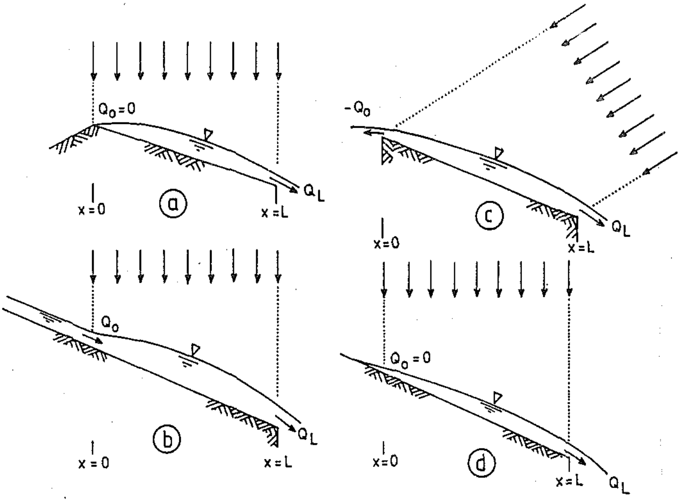

FIG. 1. Upper Boundary Conditions Commonly Used for Overland Flow

tension forces on small slope gradients and shallow flows shortly after the start of the rainfall can be considerable.

LABORATORY SETUP

The laboratory setup was composed mainly of two units:

1. A channel with a uniform rectangular cross section 1-m wide. The surface was impermeable (smooth concrete).
2. A rainfall simulator, described in Lima (1989b), consisting of circular plates and conveyors extending to the edges of the plates, where the drops are formed. The simulator generates a uniformly distributed and time-invariant geometrical rainfall pattern over a length L measured along the plane (from $x = 0$ to $x = L$; $x = 0$ is defined as the upslope limit of the applied rainfall).

Primary data were collected at steady state for each simulation run (defined as the experiment conducted with fixed length, slope, and rainfall intensity). They were length L, slope S, rainfall intensity P, water temperature, mean depth of flow at flume outlet h_L, and backwater distance measured upslope of $x = 0$ D (see Fig. 2). The experiments were repeated for combinations of lengths ($L = 3.27$ m and 4.72 m), slope gradients (ranging from 0.1% to 4%), and rainfall intensities (ranging from 0.0207 to 0.1528 mm/s).

The measuring procedure was the following: (1) Start initial rainfall on dry surface; (2) wait until equilibrium (steady state) is reached; (3) measure primary data; and (4) increase or decrease discharge and repeat steps 2–4. It was possible to install a splash barrier at $x = 0$ [Fig. 2(a)] to prevent splash droplets from striking the surface upslope of $x = 0$.

RESULTS

The overland-flow sheet observed at steady state during the experimental runs (both with and without the upstream splash barrier) could be divided into the following sections [Figs. 2(*a*) and (*b*)]:

Section 1

Highly disturbed, downslope overland flow with direct impact of raindrops and raindrop splash droplets. Dye injected into the raindrops and the overland-water sheet was rapidly dispersed.

Section 2a

Disturbed, mainly radial flow with small circular wave formation owing to drop impact in the first section.

Section 2b

Disturbed, mainly radial flow with small circular wave formation owing to drop impact in the first section. Direct impact of splash droplets also existed.

Section 3

Stagnant water (horizontal water surface). After injection of dye, no water movement was observed (except for gradual dye diffusion).

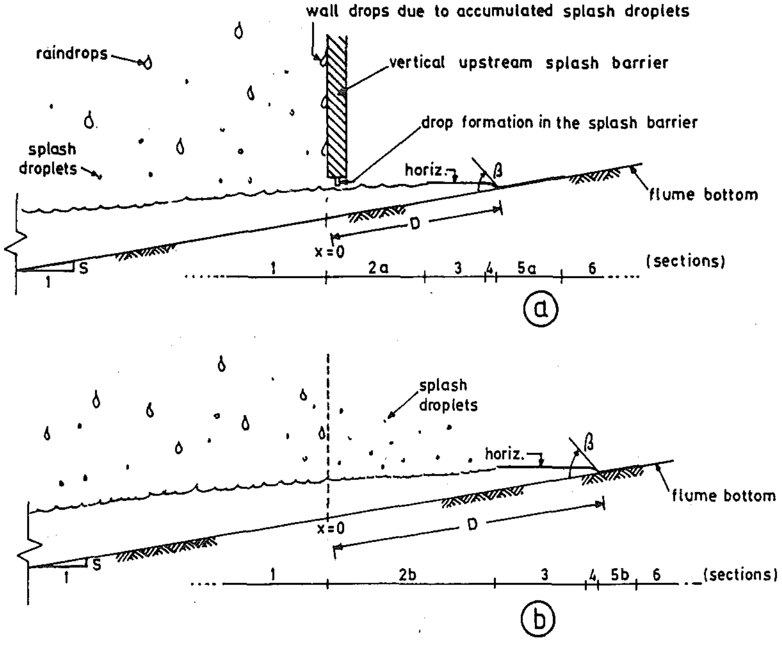

FIG. 2. Observed Sections on Overland Flow Sheet: (*a*) with Splash Barrier; and (*b*) without Splash Barrier

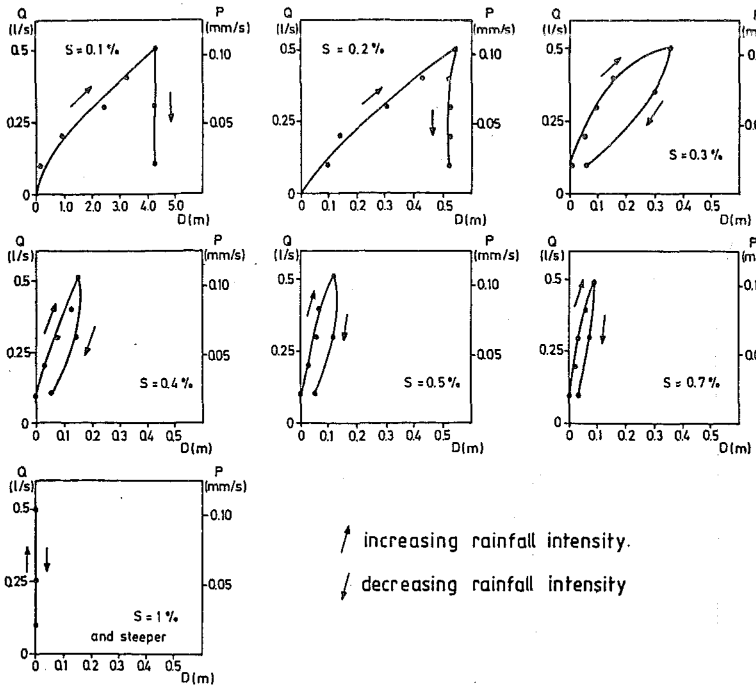

FIG. 3. Hysteresis Effect in Rainfall Intensity P-Backwater Distance D Relationship for Different Slopes S

Section 4

Adhesive water. Solid-liquid tensions with a contact angle β were present at the bottom of the flume surface.

Section 5a

Wetted flume bottom (prewetted with a moistened cloth).

Section 5b

Wetted flume bottom (prewetted with a moistened cloth) with scattered water bubbles formed by the splash droplets.

Section 6

Air-dry flume bottom.

Rainfall intensity plotted against the backwater distance, for a fixed slope, was strongly subjected to hysteresis because of surface tension effects on the flume bottom (Fig. 3). The hysteresis effect decreased as the slope gradient increased. For the 0.1% slope, the backwater effect was pronounced, with D values exceeding 4.0 m.

The removal of the splash barrier increased D (Fig. 2). Apparently, the length L of the rainfall application had no effect on D for either of the two lengths used. The importance of surface-tension and wetness characteristics in the feasibility of scale modeling of the rainfall-surface runoff process on

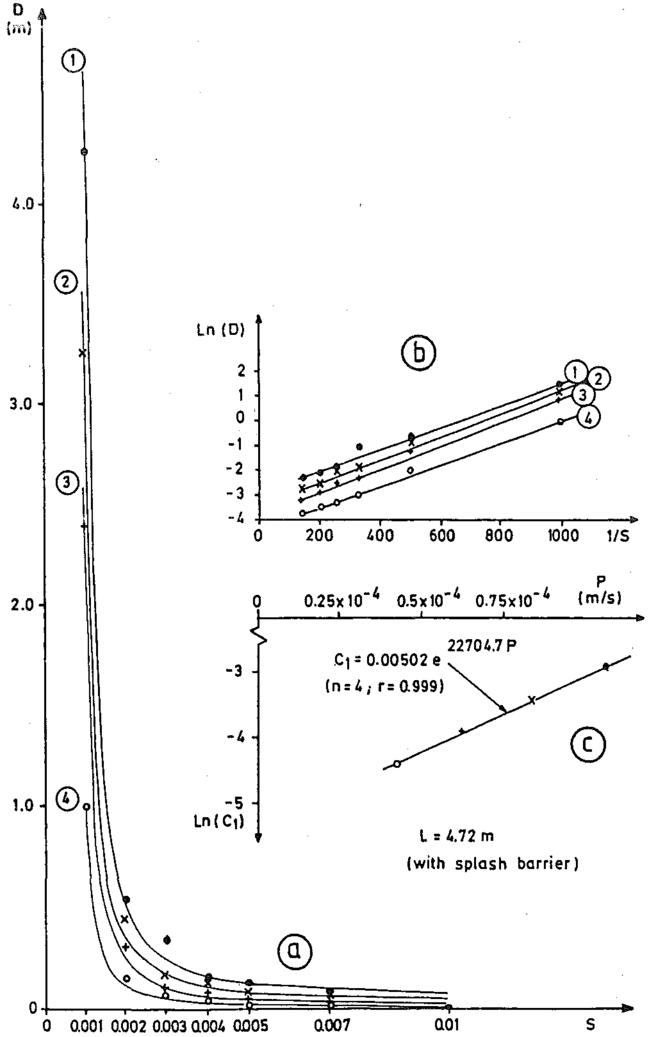

FIG. 4. (*a*) *D*-*S* Relationship in Linear Scales; (*b*) ln (*D*) against 1/*S*; and (*c*) ln (C_1) against *P*

impermeable planes has been investigated by Graveto (1970).

In Fig. 4(*a*), *D* was plotted against the slope in linear scales for different rainfall intensities and for $L = 4.72$ m. For a horizontal surface ($S = 0$), *D* tends to infinity at steady state. In Fig. 4(*b*), ln (*D*) was plotted against 1/*S* for the same rainfall intensities. Linear relations were fitted with high-regression coefficients. Thus, for a certain rainfall intensity, *D* was estimated as a function of the slope by

$$D = C_1 e^{C_2/S} \quad (1)$$

TABLE 1. Parameters C_1 and C_2 of Eq. 1 for Curves of Fig. 4

Curve number (of Fig. 4) (1)	Discharge at $x = L$ (m^3 s^{-1}) (2)	Rainfall P (mm s^{-1}) (3)	Parameters of Eq. 1 C_1 (m) (4)	Parameters of Eq. 1 C_2 (−) (5)	Regression coefficient r (−) (6)
1	0.00051	0.106	0.0548	0.00444	0.990
2	0.00040	0.083	0.0333	0.00467	0.994
3	0.00030	0.062	0.0213	0.00480	0.994
4	0.00020	0.042	0.0127	0.00441	0.997

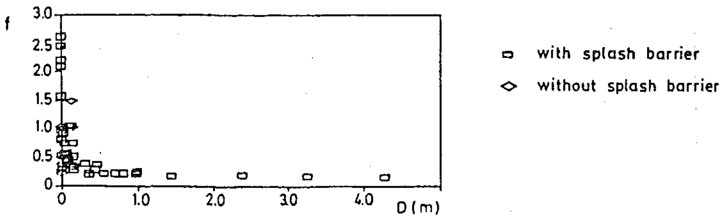

FIG. 5. Relationship between f and D

where D = the backwater distance (m); and C_1 (m) and C_2 (−) = parameters. Values of C_1 and C_2 are presented in Table 1.

C_2 can be considered independent of slope and rainfall intensity because of the approximately parallel lines in Fig. 4(b). Therefore, the plot of ln (C_1) against P results in a straight line, defined by [Fig. 4(c)]

$$C_1 = C_3 e^{C_4 P} \qquad (2)$$

where C_3 (m) and C_4 (m^{-1} s) = parameters; and P = the rainfall intensity (m s^{-1}).

Substitution of Eq. 2 into Eq. 1 yields an expression for D as a function of S and P:

$$D = C_3 e^{(C_4 P + C_2/S)} \qquad (3)$$

Parameters C_2, C_3, and C_4 were calculated by regression techniques applied to the laboratory data (smooth concrete surface): $C_2 = 0.00458$; $C_3 = 0.00502$ m; and $C_4 = 22{,}704.7$ m^{-1} s.

In Fig. 5, the Darcy-Weisbach friction factor f was plotted against D for measurements with increasing rainfall intensities P. The following expression was used to calculate the Darcy-Weisbach friction factor, assuming that the friction slope S_f equals the surface slope S:

$$f = \frac{8 g h^3 S}{q^2} \qquad (4)$$

where f = the Darcy-Weisbach friction factor (−); g = the acceleration due to gravity (m s^{-2}); h = the flow depth at $x = L$ (m); and q = the discharge per unit width ($q = vh$), at $x = L$ (m^2 s^{-1}).

The strong reduction of the friction factor f observed in Fig. 5 for higher values of D is mainly caused by changes in rainfall intensity P and slope S.

CONCLUSIONS

For gently sloping impermeable plane surfaces under vertical rainfall, care should be taken in using $h(0,t) = 0$ for $t \geq 0$ as an upper boundary condition for overland flow. The observed nonzero water depth at $x = 0$ and the consequent increase of water depth over the surface radically change the overland flow process over the slope. The form of the f-D relationship, and indirectly, the form of the f-Re relationship, which are of fundamental importance to the mathematical modeling of overland flow, were affected. The implication of these results for overland flow modeling will be the subject of future work.

Because surface-tension forces are important factors that affect shallow flow processes, additional research could focus on the effects of roughness and surface wetness characteristics.

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