Critical Rainfall Duration for Maximum Discharge from Overland Plane

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Abstract: By applying the kinematic wave method to the analysis of overland flow through a rectangular plane, a functional relationship between the critical rainfall duration associated with the maximum peak discharge, and the characteristics of the rainfall intensity-duration curve has been formulated. The traditional approach of equating the critical duration to the time of concentration is valid for planes with low infiltration loss. In this case, the full plane area contributes to the maximum peak discharge. However, for planes with high infiltration loss, the duration of the critical rainfall associated with the maximum peak discharge may be shorter than the time of concentration. Under this condition, only part of the plane contributes to the maximum peak discharge. A method for determining the critical rainfall duration and equations for determining the maximum peak discharge have also been developed.

INTRODUCTION

For drainage design of a small basin without detention storage, the peak discharge of the storm runoff is traditionally estimated based on the rainfall intensity whose duration equals the time of concentration of that basin. This design concept associates the peak discharge with the rainfall-runoff process under which the entire basin is contributing. Based on the observation that the area contributing to direct runoff was usually a fraction of the basin area, Betson (1964) introduced the concept of partial-area contribution. Several researchers (Larson and Machmeier 1968; Meynink and Cordery 1976; Bondelid and McCuen 1979; Stephenson and Meadows 1986) also found that the critical duration of rainfall that caused the maximum peak discharge could be shorter than the basin time of concentration. Therefore, partial-area contribution could produce the maximum peak discharge.

This paper examines the functional relationship between the critical rainfall duration and the characteristics of the rainfall intensity-duration curve, and develops a method on determining the critical rainfall duration and the maximum peak discharge.

TIME OF CONCENTRATION

By applying the kinematic wave equations to flow over an overland plane with an initially dry surface due to a uniform rainfall excess, Chen and Wong (1990) showed that the time of concentration, t_o, could be determined by

$$t_o = \left(\frac{0.21L_o^2 f}{Si_n}\right)^{1/3} \quad \text{(1)}$$

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Note. Discussion open until February 1, 1994. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on April 27, 1992. This paper is part of the ***Journal of Hydraulic Engineering***, Vol. 119, No. 9, September, 1993. ©ASCE, ISSN 0733-9429/93/0009-1040/$1.00 + $.15 per page. Paper No. 3944.

where L_o = length of the overland plane; f = Darcy-Weisbach resistance coefficient; S = slope of the overland plane; and i_n = net rainfall intensity. In (1), the units are min for t_o, m for L_o, mm/hr for i_n, and both S and f are dimensionless. Further, the coefficient, f, can be related to the Reynolds number, R, as follows:

$$f = \frac{C}{\mathsf{R}^k} \qquad (2)$$

where C and k = constants. The Reynolds number, R, is related to the flow at the end of the plane at equilibrium

$$\mathsf{R} = \frac{1}{K}\left(\frac{i_n L_o}{\nu}\right) \qquad (3)$$

where $K = 3.6 \times 10^6$ and ν = the kinematic viscosity of water (m²/s). Substituting (2) and (3) into (1), the time of concentration expression becomes

$$t_o = \left[\frac{0.21(K\nu)^k C L_o^{2-k}}{S i_n^{1+k}}\right]^{1/3} \qquad (4)$$

By substituting $k = 0$ to 1, (14) can be applied to flow regimes from turbulent to laminar.

RAINFALL INTENSITY-DURATION RELATIONSHIP

For a given recurrence interval, the net rainfall intensity, i_n, can be related to the net rainfall duration, t_n, as follows (Chen and Evans 1977):

$$i_n = a_n t_n^{-b_n} \qquad (5)$$

where the coefficient a_n and the exponent b_n are functions of t_n. For a plane with a given infiltration rate, f_i, the net rainfall curve can be constructed from the published design rainfall intensity-duration curve by adjusting the rainfall intensity, i, to the net intensity, i_n, as follows:

$$i_n = i - f_i \qquad (6)$$

The units are mm/hr for i and f_i. The net rainfall curve can then be divided into segments, and represented by (5) with discrete values of a_n and b_n for each segment.

PEAK DISCHARGE—FULL AND PARTIAL AREA CONTRIBUTION

For full-area contribution, an explicit expression for the design rainfall intensity, i_o, can be determined by equating t_o in (4) to t_n in (5), and by substituting i_o for i_n

$$i_o = \left\{\frac{a_{no}^{1/b_{no}}}{\left[\frac{0.21(K\nu)^k C L_o^{2-k}}{S}\right]^{1/3}}\right\}^{3b_{no}/[3-(1+k)b_{no}]} \qquad (7)$$

where a_{no} and b_{no} = respective values of a_n and b_n for $i_n = i_o$ and $t_n = t_o$. By substituting i_o into $q_o = i_o L_o / K$, the peak discharge per unit width of the plane, q_o in m²/s, can be expressed as

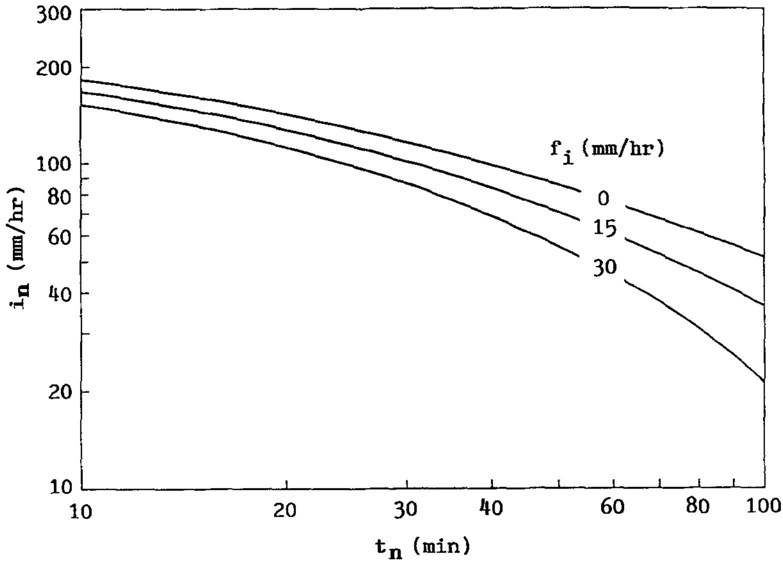

FIG. 1. Singapore Five-Year Rainfall Intensity-Duration Curves—with and without Infiltration

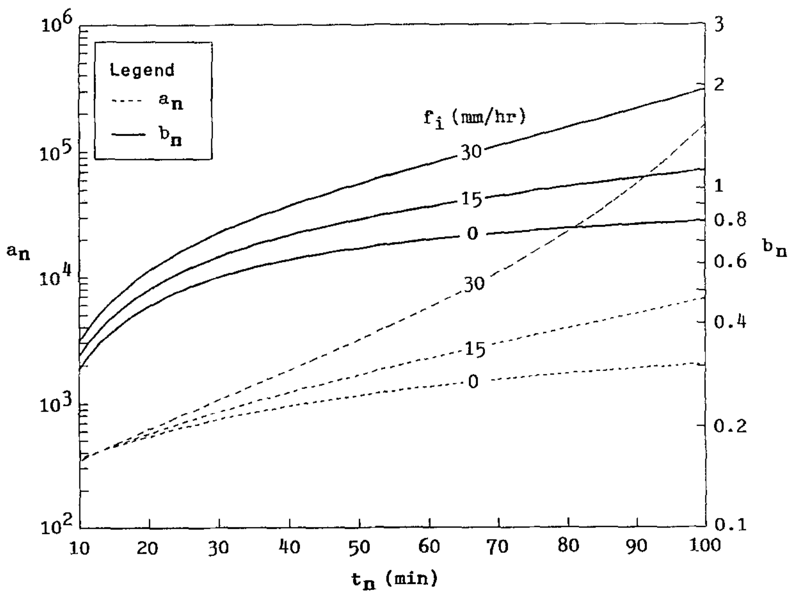

FIG. 2. Variation of a_n and b_n with Rainfall Duration

$$q_o = \frac{1}{K}\left\{\frac{a_{no}^{1/b_{no}} L_o^{(1-b_{no})/b_{no}}}{\left[\frac{0.21(K\nu)^k C}{S}\right]^{1/3}}\right\}^{(3b_{no})/[3-(1+k)b_{no}]} \quad \text{(8)}$$

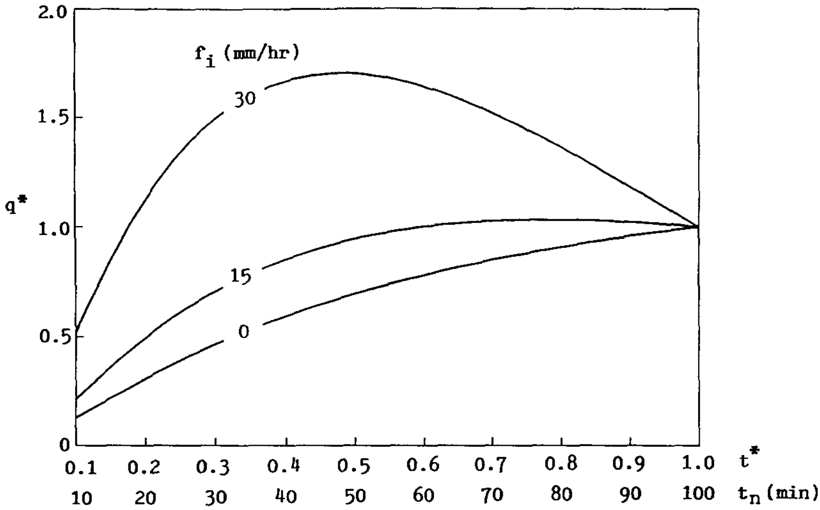

FIG. 3. Variation of Discharge Ratio with Time Ratio

Similarly, for a plane of length L_p (where $L_p < L_o$), the time of concentration t_p (where $t_p < t_o$) can be determined from (4) by substituting t_p for t_o and L_p for L_o. Further, the design rainfall intensity i_p, and the unit peak discharge q_p can be determined from (7) and (8), respectively, by substituting i_p for i_o, a_{np} for a_{no}, b_{np} for b_{no}, L_p for L_o, and q_p for q_o.

Based on the kinematic wave method, the equations for t_p, i_p, and q_p are equally applicable to a plane of length L_o, subject to a storm of net duration t_p (where $t_p < t_o$). In this case, the portion of the plane contributing to q_p is L_p (where $L_p < L_o$).

COMPARISON OF PEAK DISCHARGES

Fig. 1 shows the five-year rainfall intensity-duration curve of Singapore (*Code*, Singapore 1991) and the corresponding net rainfall curves for f_i = 15 and 30 mm/hr. Fig. 2 shows the corresponding values of a_n and b_n as defined in (5). In order to compare the peak discharges from the full-area and partial-area contribution, the three net rainfall curves were applied to grass planes with a longitudinal slope $S = 1\%$. Values of $C = 400$, $k = 0.5$, and $\nu = 10^{-6}$ m^2/s were assumed in the illustration. For an arbitrarily chosen time of concentration of 100 min, the lengths of the planes, L_o, were estimated from (4), as 804 m for $f_i = 0$ mm/hr, 568 m for $f_i = 15$ mm/hr, and 333 m for $f_i = 30$ mm/hr.

The comparison of peak discharges was carried out by means of the dimensionless discharge ratio, $q^* = q_p/q_o$, and time ratio, $t^* = t_p/t_o$. As shown in Fig. 2, for the rainfall curve with no infiltration, the value of b_n is less than unity for all durations. Fig. 3 shows that q^* increases with increasing t^*, and reaches a maximum value at $t^* = 1$. Therefore, the peak discharge occurs under the condition of full-area contribution. For this case, the critical duration is the time of concentration, t_o, and the peak discharge, q_o, determined from (8) is the maximum peak discharge.

For the rainfall curves with f_i = 15 and 30 mm/hr, the value of b_n exceeds unity at $t_n > 78$ and 48 min, respectively (Fig. 2). The net duration at b_n

= 1 is defined as t_u. As shown in Fig. 3, q^* is maximum at $t^* = 0.78$ for $f_i = 15$ mm/hr, and at $t^* = 0.48$ for $f_i = 30$ mm/hr. These critical durations correspond to the duration t_u of the respective rainfall curve. Since the critical durations are shorter than the time of concentration $t_o = 100$ min, the maximum peak discharges in fact occur under the condition of partial-area contribution. The maximum peak discharge, q_u, can be determined from the q_p equation directly by setting $b_{np} = 1$

$$q_u = \frac{1}{K}\left[\frac{a_{nu}}{\left\{\frac{0.21(K\nu)^k C}{S}\right\}^{1/3}}\right]^{3/(2-k)} \quad (9)$$

where a_{nu} = the value of a_n at $b_n = 1$.

CONCLUSIONS

For a homogeneous, rectangular overland plane, the duration of the critical rainfall that produces the maximum peak discharge is the smaller of (1) The time of concentration of the plane, t_o; or (2) the net duration, t_u, at which the exponent b_n in (5) is unity. The maximum peak discharge can be determined from (8) if the critical duration is t_o, or from (9) if the critical duration is t_u.

APPENDIX I. REFERENCES

Betson, R. P. (1964). "What is watershed runoff?" *J. Geophysical Res.*, 69(8), 1541–1552.

Bondelid, T. R., and McCuen, H. (1979). "Critical storm duration for the rational method." *J. Civ. Engrg. Design*, 1(3), 273–285.

Chen, C. N., and Evans, R. R. (1977). "Application of kinematic wave method to predict overland peak flows." *Proc. Int. Symp. on Urban Hydrology, Hydraulics and Sediment Control*, Univ. of Kentucky, Lexington, Kentucky, 113–118.

Chen, C. N., and Wong, T. S. W. (1990). "Evaluation of resistance coefficients for overland flow analysis." *Drainage Systems and Runoff Reduction: Proc. 5th Int. Conf. on Urban Storm Drainage*, Y. Iwasa and T. Sueishi, ed., Vol. 2, Osaka Univ., Osaka, Japan, 1059–1064.

Code of practice on surface water drainage. (1991). Ministry of the Envir., Drainage Dept., Singapore.

Larson, C. L., and Machmeier, R. E. (1968). "Peak flow and critical duration for small watersheds." *Trans. Am. Society of Agric. Engrg.*, 11(2), 208–213.

Meynink, W. J. C., and Cordery, I. (1976). "Critical duration of rainfall for flood estimation." *Water Resour. Res.*, 12(6), 1209–1214.

Stephenson, D., and Meadows, M. E. (1986). *Kinematic hydrology and modelling.* Elsevier, Amsterdam, the Netherlands.

APPENDIX II. NOTATION

The following symbols are used in this paper:

a_n = coefficient in (5);
a_{no} = a_n at $t_n = t_o$;
a_{np} = a_n at $t_n = t_p$;
a_{nu} = a_n at $b_n = 1$;
b_n = exponent in (5);
b_{no} = b_n at $t_n = t_o$;

b_{np} = b_n at $t_n = t_p$;
C = constant in (2);
f = Darcy-Weisbach resistance coefficient;
f_i = infiltration;
i = rainfall intensity;
i_n = net rainfall intensity;
i_o = net rainfall intensity at $t_n = t_o$;
i_p = net rainfall intensity at $t_n = t_p$;
K = 3.6×10^6;
k = constant in (2);
L_o = length of overland plane with time of concentration t_o;
L_p = length of overland plane with time of concentration t_p;
q_o = peak discharge per unit width of overland plane for $t_n = t_o$;
q_p = peak discharge per unit width of overland plane for $t_n = t_p$;
q_u = peak discharge per unit width of overland plane for $t_n = t_u$;
q^* = q_p/q_o;
R = Reynolds number;
S = slope of overland plane;
t_n = net rainfall duration;
t_o = time of concentration for plane with length L_o;
t_p = time of concentration for plane with length L_p;
t_u = t_n at $b_n = 1$;
t^* = t_p/t_o; and
ν = kinematic viscosity of water.