S., Evaporation from the 17 weste
.th a section on evaporation rates b
ɪrdenson, U.S. Geol. Surv. Prof. Pa
-100, Washington, D. C., 1962.
M. J., and G. L. Bodhaine, Investiga
ɪe rise in level of Soap Lake at Soap
ɪshington, U.S. Geol. Surv. Open Fil
117, Tacoma, Washington, 1954.
E., Lakes of Washington, Bull. Wat
, vol. 2, Eastern Washington, 650 pp
of Water Resources, Olympia, Wash
64.

script received October 5, 1970;
revised March 4, 1971.)

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Overland Flow on an Infiltrating Surface

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Abstract. The partial differential equation for vertical, one-phase, unsaturated moisture flow in soils is employed as a mathematical model for infiltration rate. Solution of this equation for the rainfall-ponding upper boundary condition is proposed as a sensitive means to describe infiltration rate as a dependent upper boundary condition. A nonlinear Crank-Nicholson implicit finite difference scheme is used to develop a solution to this equation that predicts infiltration under realistic upper boundary and soil matrix conditions. The kinematic wave approximation to the equations of unsteady overland flow on cascaded planes is solved by a second order explicit difference scheme. The difference equations of infiltration and overland flow are then combined into a model for a simple watershed that employs computational logic so that boundary conditions match at the soil surface. The mathematical model is tested by comparison with data from a 40-foot laboratory soil flume fitted with a rainfall simulator and with data from the USDA Agricultural Research Service experimental watershed at Hastings, Nebraska. Good agreement is obtained between measured and predicted hydrographs, although there are some differences in recession lengths. The results indicate that a theoretically based model can be used to describe simple watershed response when appropriate physical parameters can be obtained.

The overland flow and rainfall infiltration phases of hydrology have been studied extensively as separate systems [*Woolhiser and Liggett*, 1967; *Kibler*, 1968; *Whisler and Klute*, 1965; *Hanks and Bowers*, 1962; *Philip*, 1957; *Rubin*, 1966]. Although physically based overland flow models have been combined with simplified lumped-system infiltration models [*Wooding*, 1965; *Burman*, 1969; *Foster*, 1968], they have not as yet been combined with an infiltration model derived from soil moisture flow theory. The latter combination represents a first approximation to a physically based theoretical model of an infiltrating watershed, and the purpose of this paper is to describe such a model and to present results from laboratory and field experiments designed to verify the model.

THE INFILTRATION MODEL

Water movement in unsaturated soil may be described by equations of two-phase porous media flow in which air is the second fluid phase. For vertical, one-dimensional flow, when the air phase can be assumed to move under negligible pressure gradients, moisture movement can be described by Richard's equation,

$$\frac{\partial}{\partial t}(\phi S_w) = K\left[\frac{\partial}{\partial z}\left(k_r \frac{\partial \psi}{\partial z}\right) - \frac{\partial k_r}{\partial z}\right] \quad (1)$$

where ϕ is the porosity, S_w is the volumetric saturation, K is the hydraulic conductivity, k_r is the relative permeability, ψ is the water pressure potential, and z and t are the spatial and time coordinates, respectively. Solution of this equation requires knowledge of the relation between S_w and ψ (moisture-tension curve, $S_w = S_w(\psi)$) and the relation between k_r and ψ (relative permeability curve, $k_r = k_r(\psi)$). The simplifying assumptions concerning physical conditions on which (1) is based have been discussed extensively elsewhere [*Smith*, 1970; *Freeze*, 1969].

For moisture flow from rainfall infiltration, the appropriate upper boundary conditions are: (1) $t_o < t < t_p$, $f(0, t) = r(t)$ (rainfall rate), where t_o is the starting time, t_p is the time at which $S_w(0, t)$ reaches the maximum value (saturated equals one) or $\psi(0, t) = 0$, and $f(0, t)$ is the moisture flux into the soil surface; (2) $t > t_p$, $S_w(0, t) = 1$ or $\psi(0, t) = h(x, t)$, where $h(x, t)$ is the surface water depth at location x.

The initial conditions may be any physically realizable array of pressures, $\psi(z, 0) = \psi_o$, $0 < z < L$. The lower boundary condition may be a stable water table at some depth L such that $\psi(L, t) = 0$, $0 < t < \infty$.

The solution surface for equation 1 under these conditions is illustrated in Figure 1, in which S_w is shown as a function of z and t. For mathematical reasons the equation is actually solved for $\psi(z, t)$ and converted by the $S_w(\psi)$ relation, since S_w is often of greater interest.

The more simple horizontal flow version of equation 1 (when the last term is equal to zero) cannot be solved by analytical methods without special simplifying assumptions [*Klute*, 1952; *Philip*, 1957]. The greatest difficulty comes from the complex nature of the functions $k_r(\psi)$ and $S_w(\psi)$. Vertical unsaturated flow is mathematically even more difficult. Thus finite difference techniques are commonly employed for its solution [*Hanks and Bowers*, 1962; *Whisler and Klute*, 1965; *Rubin and Steinhardt*, 1963]. Most investigators who have published solutions to this equation have been concerned with determining moisture profiles rather than infiltration rates, and assumptions used in the finite difference method reflect this purpose. Linearization of the finite difference equations and selection of z and t increment sizes are important in this regard [*Smith*, 1970].

The infiltration rate $f(t)$ may be computed from a solution of equation 1 in terms of $S_w(z, t)$ by two methods. First,

$$f(t) = \int_{z=0}^{z=L} \phi(z) \frac{dS_w}{dt}(z, t)\, dz \qquad (2)$$

For $t_o < t < t_p$, $f(t) = r(t)$. When $t > t_p$, however, $f(t) < r(t)$. In either case, $f(t)$ should be equal to the flow rate at $z = 0$ as computed by Darcy's law:

$$f(t) = -Kk_r(0, t)\left[\frac{\partial}{\partial z}\psi(0, t) - 1\right] \qquad (3)$$

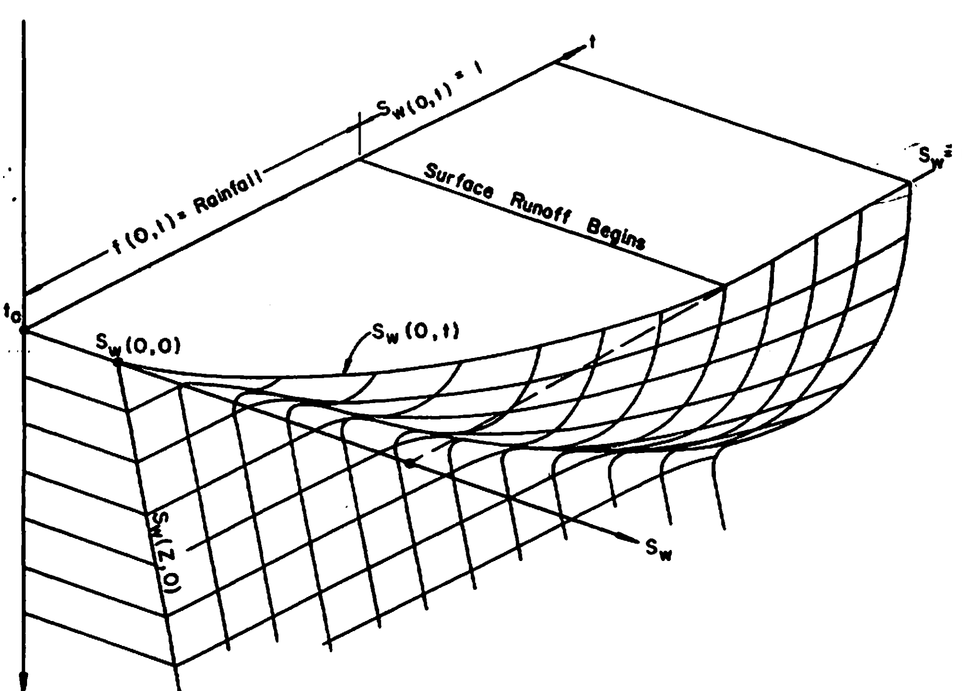

Fig. 1. Solution surface in (S_w, z, t) for equation 1.

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THE OVERLAND FLOW MODE

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$$\frac{\partial h}{\partial t} + \alpha(m+1)h^m \frac{\partial h}{\partial x} = q(x$$

where $h(x, t)$ is the surface depth,
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The derivation and applications
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COMBINED MODEL OF OVERLAN
AND INFILTRATION

The simplified watershed mode
here may consist of a cascade

Comparison of the $f(t)$ computed by the two methods provides a means to evaluate the accuracy of the solution scheme. This is a sensitive comparison that many difference methods fail to satisfy [*Smith*, 1970].

THE OVERLAND FLOW MODEL

It has been shown [Woolhiser and Liggett, 1967] that the equations of nonuniform unsteady flow may be simplified to the kinematic wave equation for most overland flow situations of hydrologic significance:

$$\frac{\partial h}{\partial t} + \alpha(m+1)h^m \frac{\partial h}{\partial x} = q(x, t) \qquad (4)$$

where $h(x, t)$ is the surface depth, $q(x, t) = r(t) - f(x, t)$, x is the distance along the slope, and $m = \frac{1}{2}$ (for turbulent flow) or 2 (for laminar flow). The coefficient α is evaluated from the Darcy-Weisbach relationship,

$$V^2 = 8gRS/f \qquad (5)$$

where R is the hydraulic radius, g is the gravitational acceleration, and f is the Darcy-Weisbach friction factor. Under turbulent flow, if one assumes that $f = k_t$ (a constant), equation 5 reduces to $V = C_t(RS)^{1/2}$ and C_t is the Chézy coefficient. Under laminar flow, if one assumes that $f = k_1/N_r$ (where k_1 is a constant greater than 24 and N_r is the Reynolds number), equation 5 reduces to $V = C_1\, SR^2$. When $R \simeq h$, one may generalize (5) to $V = \alpha h^m$, where $\alpha = CS^{(m+1)/2}$.

The derivation and applications of equation 5 are discussed elsewhere [*Wooding*, 1965; *Kibler and Woolhiser*, 1970; *Smith*, 1970]. Like the soil moisture flow equation, this partial differential equation is solved by finite difference techniques.

The initial conditions specify a dry surface; e.g., $h(x, t) = 0$ at $t = 0$ for all x. Boundary conditions are determined by the geometry; i.e., at the uppermost end of a watershed $h(0, t) = 0$ and for the junction of two planes, the continuity of flow is preserved on the basis of computed flow from the upper plane.

COMBINED MODEL OF OVERLAND FLOW AND INFILTRATION

The simplified watershed model considered here may consist of a cascade of planes of

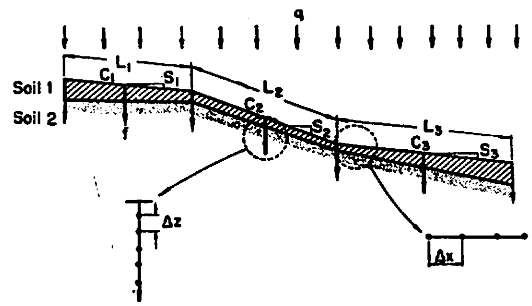

Fig. 2. Schematic representation of the mathematical watershed model.

different slope, width, roughness, and length, as shown in Figure 2. The soil may be layered, and layering may vary in any uniform manner between plane intersections. The mathematical model of such a watershed consists of simultaneous solution of finite difference formulations of equations 1 and 4. Solution is simultaneous in the sense that solutions move concurrently in time, boundary conditions being interdependent. The soil moisture flow equation is solved at as many points along the surface as are necessary to define the horizontal variation in soil properties. The z and x dimensions are divided into finite increments to solve equations 1 and 4, respectively (Figure 2). Equation 1 is solved by using an implicit nonlinear Crank-Nicholson finite difference formulation, presented in the appendix. Solution of equation 4 employs an explicit difference method known as the single-step Lax-Wendroff scheme. Derivation of this finite difference method is discussed elsewhere [*Kibler and Woolhiser*, 1970]. Values of $q(x, t)$ are provided at each t by solution of equation 3 in finite difference form, since $q(x, t) = r(t) - f(x, t)$.

The Crank-Nicholson implicit difference scheme was chosen after comparison of several difference methods on the basis of stability, accuracy, and preservation of material continuity. For each time step the finite difference form of equation 1 is reduced to a set of nonlinear equations in $\psi(z, t)$ for the end of the time step, and the resulting matrix equation is solved by Jacobi iteration.

Sizes of Δt and Δz increments are an important factor in the accuracy of the finite difference solution. The effects of increment sizes are discussed elsewhere [*Smith*, 1970]. As the solution proceeds, Δt is determined such that the curvilinear nature of the $S_w(\psi)$ and $k_r(\psi)$ relations is approximated by movement along arbitrarily small chords.

A logical algorithm is used to detect the current state of the watershed surface for proper assignment of boundary conditions for the new time $t_2 = t_1 + \Delta t$, at which equation 1 is being solved for the array of ψ. When ψ at the surface is close to 0, it is necessary to know whether the soil is saturating or desaturating. If the surface is saturating, it may be necessary to reduce Δt so that the point in time at which $\psi(0, t) = 0$ is found closely enough to provide a smooth transition of boundary conditions. If the surface water depth is receding, it is necessary to investigate conditions to see if available surface water plus rainfall is less than potential infiltration, so that the upper boundary condition may be reset to the unsaturated case for time t_2.

LABORATORY AND FIELD EXPERIMENTS

To gain insight into the sensitivity of the watershed model to input parameters and to study its applicability and efficiency as a research tool, the model was used to predict results from two significantly different experimental watersheds.

Simulation of a Laboratory-Scale Watershed

To determine the performance of the model in predicting surface runoff, infiltration, and soil moisture movement processes, a laboratory-scale soil flume was modified to create a prototype infiltrating slope. The flume is 40 feet long, 2 inches wide, and 4 feet deep; the sides are made of 4- by 4-foot, ¾-inch panels of aluminum or plexiglass. The ends of the flume were made porous to collect seepage flow, and the slope of the flume may be changed by a hydraulic lift.

To prevent algal growth in the porous medium, the fluid used is a light oil resembling refined kerosene that the petroleum industry uses as a core test fluid. Its viscosity is close to that of water, and its capillary properties were determined by laboratory measurement by using the soil in the flume. This soil is a locally obtained river-deposited sand known as Poudre fine sand. Although care was taken in placing the soil, uniform density was not achieved.

To provide simulated rainfall, drop-producing manifolds were constructed similar to those described by *Chow and Harbaugh* [1965]. The soil surface was covered with gauze to prevent splash erosion. Runoff was collected and the rate measured continuously by a pressure transducer connected to a collection tank. Gamma ray attenuation was employed to follow the rapid vertical movement of soil moisture. The experimental arrangement is illustrated in Figure 3.

The experimental procedure consisted of (1) determining the hydraulic properties of the flume soil, (2) determining the 'rainfall rate' for a given manifold pressure by covering the soil with a collector flume, and finally (3) simulating a rainfall event under measured initial soil conditions.

The bulk density ρ_b of the soil was measured carefully at four sections by gamma attenuation, and hydraulic properties for three densities representing the range as measured were determined in the laboratory. Curves of $S_w(\psi)$ and $k_r(\psi)$ as determined in these tests are shown in Figures 4 and 5. In the higher pressure range the results agree well with the theory of *Brooks and Corey* [1964], which specifies that

$$\frac{S_w - S_r}{1 - S_r} = S_e = \left(\frac{\psi_b}{\psi}\right)^{\lambda} \tag{6}$$

where S_r is 'residual saturation,' an empirical value, and

$$k_r = (\psi_b/\psi)^{\eta} \tag{7}$$

where ψ_b is the hypothetical 'bubbling pressure potential,' or S_e intercept, and λ and η are constants such that $\eta = 2 + 3\lambda$.

Experiments were performed using rainfalls of 15 minutes each at a rate roughly 2–3 times the saturated conductivity of the soil. A wet and a dry initial soil condition were simulated and measured by the gamma attenuation apparatus.

Soil properties as determined experimentally were used in the numerical model. The parameters α and m in equation 4 were determined by a graphical solution of the uniform flow characteristic equation obtained from this equation [*Smith*, 1970; *Kibler and Woolhiser*, 1970].

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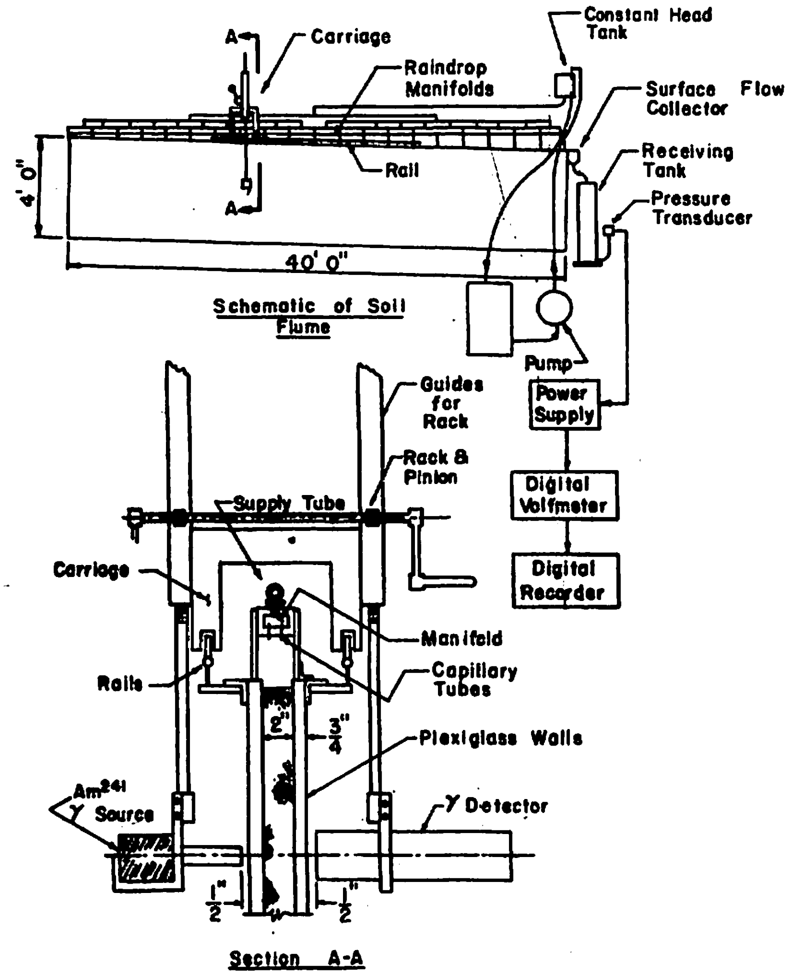

Fig. 3. General schematic of the laboratory soil flume and instrumentation for studying watershed response.

In this manner $q(x, t)$ as obtained from the soil simulation was graphically compared with $Q(t)$, the plane outflow hydrograph, and it was shown that a laminar flow relation fit the data quite well; i.e., $m = 2$.

Simulation of a Field Plot Watershed

Modeling a field plot watershed provides an experiment significantly different from the laboratory prototype scale simulation. The slope of the watershed is not the same at each point, the surface roughness and soil properties possess areal variations, and the rainfall and runoff data available are not as accurate as laboratory measurements.

The purpose of simulating the runoff response of a field plot watershed to rainfall is to determine the sensitivity of the simulation model to the above-mentioned heterogeneities and to see if approximate information concerning the unsaturated hydraulic properties of the soil can be used to predict the observed runoff within acceptable limits of error.

Selection of a field plot. The hydraulic properties of the soils were obtained from an Agricultural Research Service (ARS) publication [*Holtan et al.*, 1968] listing results of extensive sampling of ARS watersheds. From these watersheds it was desired to select a field plot with soil as uniform as possible. Furthermore the plot should be as near as possible to a point from which sampled soil data were available. It was also desired to avoid clay soils, which would be subject to cracking.

The site chosen was field plot 56-H in the watershed at Hastings, Nebraska, where experimentation has since been terminated. The soil type is Colby silt loam, and the plot is unfurrowed natural pasture 300 feet long and 100 feet wide with native grass vegetation. Plot 56-H is next to a continuously recording rain gage, and the contour maps available for this area indicate quite uniform overall slope.

The ARS soil data available included hydraulic properties for moisture desaturation at only six points on the $S_w(\psi)$ curve, and at least two of these points were in the extremely high tension range. To obtain useful curves for imbibition conditions, equations 6 and 7 were used. *Brooks and Corey* [1964] indicated that the $S_w(\psi)$ curves for imbibition and desaturation were logarithmically parallel, and on this basis the imbibition $S_w(\psi)$ curve was estimated. The region at low moisture tension was drawn in by eye from experience.

Selection of a storm for simulation. Detailed rainfall and runoff data on a 1-minute incremental basis were obtained for most of the storms producing runoff on each subwatershed. Soil moisture at 1-foot increments to a depth of 4 feet was measured by volumetric sampling twice each year.

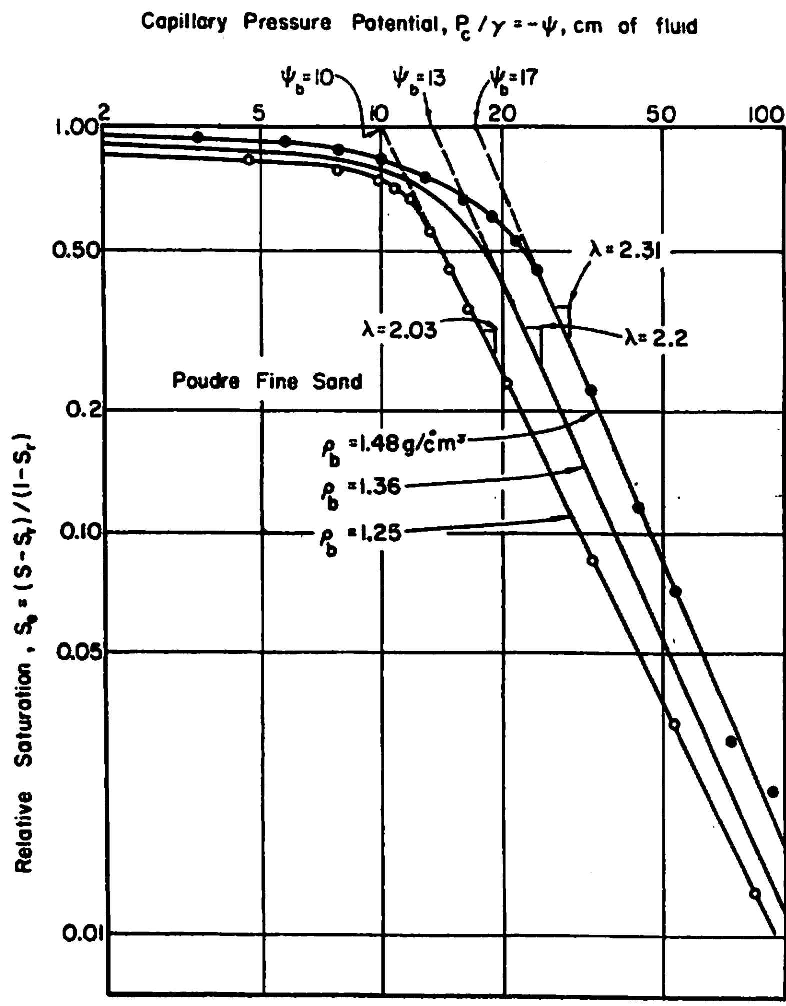

Fig. 4. Saturation–capillary pressure imbibition relations for three bulk densities of Poudre fine sand.

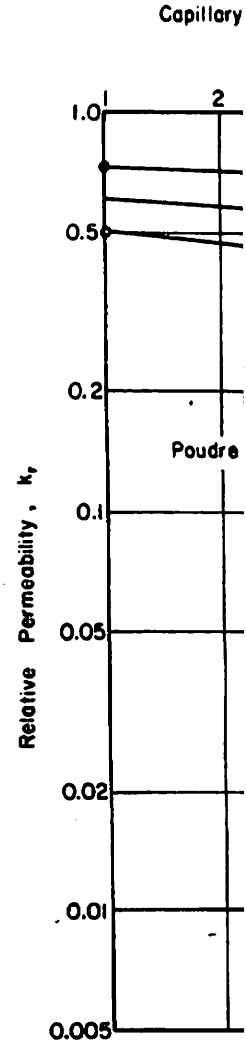

Fig. 5. Relative permeability-

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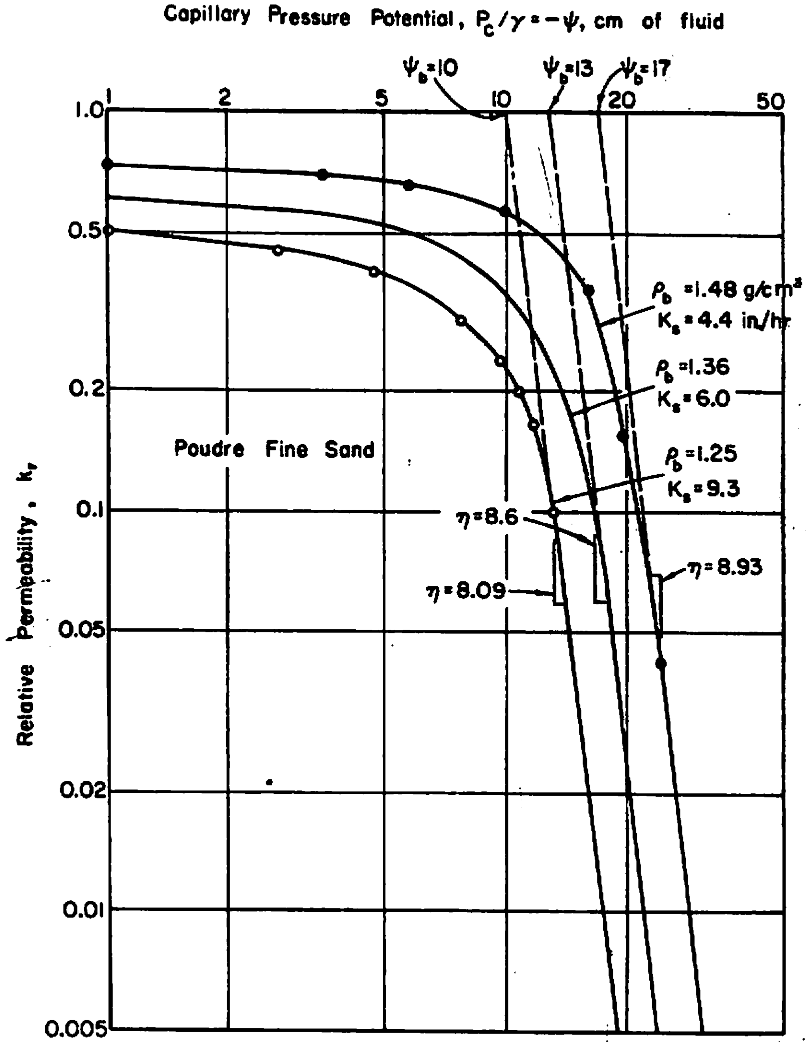

Fig. 5. Relative permeability–capillary pressure imbibition relations for three bulk densities of Poudre fine sand.

Storms were selected from these data on the basis of closeness in time to the date of soil moisture sampling and simplicity of the rainfall pattern. Unfortunately no storm met both criteria well. Either initial moisture was estimated from descriptions such as 'dry' or 'moist,' or the storm was double peaked and sufficient moisture was redistributed during the storm to make the use of the imbibition soil relations doubtful because of hysteresis.

The storms selected involved a compromise between knowledge of initial conditions and simplicity of rainfall pattern. It appears, however, that no simple storms occur in southern Nebraska.

RESULTS AND DISCUSSIONS

Simulation of the Laboratory Experiments

Five experimental runs, consisting of both 'wet' and 'dry' initial conditions, were made with the laboratory flume. The dry condition existed after the flume had drained for several weeks and the water table was approximately 42 inches deep. The wet condition existed a few hours after the run at the dry condition.

Although soil density and initial saturation were measured at four sections along the soil flume, equipment limitations prevented sampling of soil moisture movement at more than one location during an experiment. The data for this sample section, derived from gamma attenuation measurements and model simulation, are shown in Figures 6 and 7. Although soil density variations were complex, the general trend of density variation was modeled by a few layers (Figures 6 and 7). Initial measured soil moisture conditions were also simplified for use by the model.

Watershed hydrographs resulting from the experiments and the model simulation are shown in Figures 8 and 9. Corresponding saturation profiles are given in Figures 6 and 7 for the dry and wet initial conditions, respectively. The experimental results indicated a laminar flow regime throughout the hydrograph, which was used in subsequent model simulations.

Unfortunately soil density and initial saturation data were not obtainable for the upper 1½ inches of soil, owing to the construction of the flume. As a result some interpretation was involved in setting initial conditions to be used in the model. Soil densities can also be expected to vary along the flume at the surface owing to the manner in which the soil was placed in the flume. Since the manner of this variation was unknown, only two infiltrating points were used in the model simulation. Therefore some discrepancies between model and experimental results would be expected. The results nevertheless show good agreement between the model and the experiment.

Comparison of measured and simulated hydrographs consistently showed a somewhat more ex-

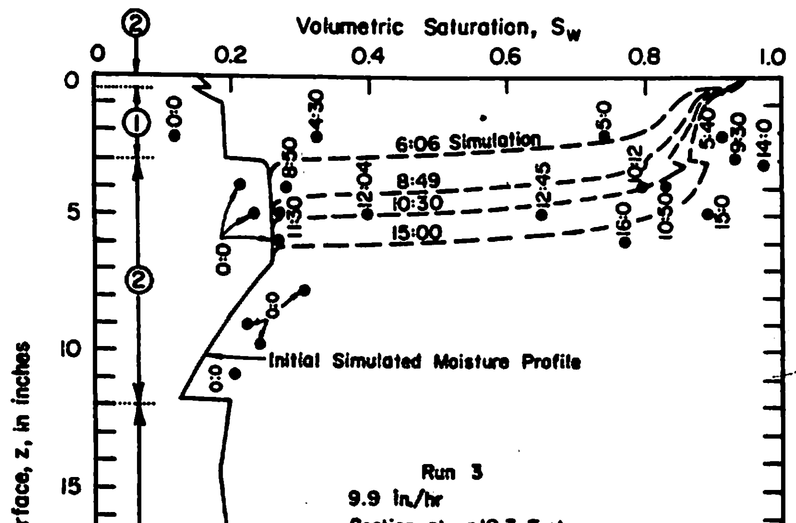

Fig. 6. Measured and simulated moisture profiles during run 3.

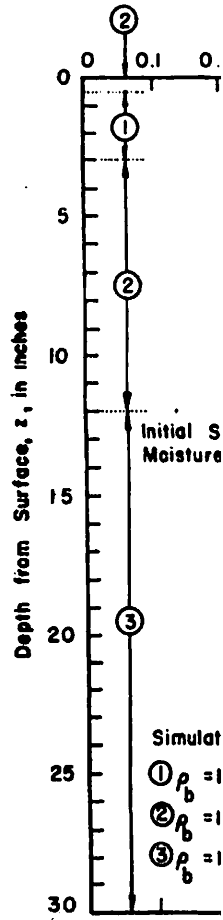

Fig. 7. Measu

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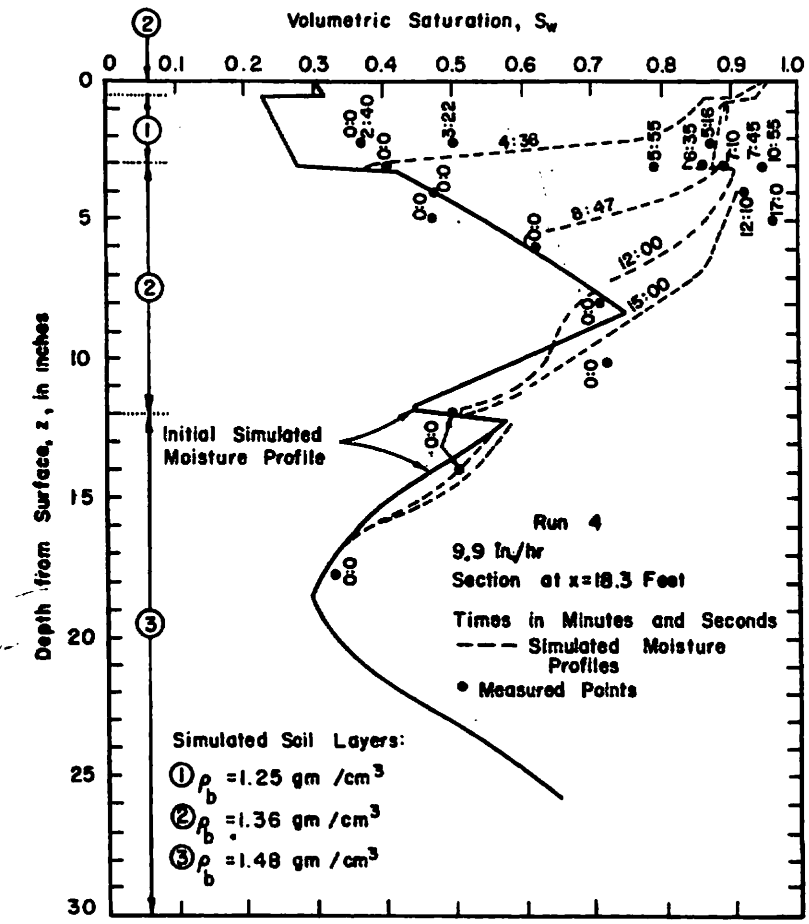

Fig. 7. Measured and simulated moisture profiles during run 4.

tended recession for the flume data than for the simulation. This disagreement could be caused by a failure of the surface flow equations to describe surface flow accurately, a failure of the infiltration model to describe infiltration, or perhaps both. For run 4 with wet initial conditions, air compression is a likely cause of the rapid jump in runoff near the end of the rainfall. This model does not account for such air counterflow. A reduction in infiltration due to air pressure gradients could perhaps account for some of the differences in recession characteristics between the experiment and the model. Bubbles were noticed escaping from the surface during the latter part of the laboratory runs.

The soil moisture profiles from the flume measurements and the corresponding results from the mathematical model (Figures 6 and 7) show a reasonable agreement. Since a minimum of 30 seconds was necessary for a significant count with the gamma attenuation equipment, the location of the steep portion of the saturation profile was obtained rather than a complete instantaneous profile. The gamma attenuation method under the conditions of this experiment could not be expected to yield data with better accuracy than 5–10%. It appears that the mathematical model overestimates the speed of the moisture 'front' by approximately 10%.

Results of Experimental Simulation of a Small Watershed Plot

Simulation of runoff from experimental plots at Hastings, Nebraska, presented a considerably different problem. All soil and hydraulic information was less detailed and accurate than that for the laboratory model. Soil imbibition curves had to be estimated from desaturation curves, for which only a few points were available. The data source gave two widely different values for saturated conductivity (0.18–0.81 in./hr). Thus saturated conductivity could not be taken as a known parameter. The roughness parameter of the surface was unknown, and the slightly undulating surface was necessarily assumed to be a plane. Furthermore the initial moisture condition of the storms could only be estimated.

With these limitations, attempts at simulating plot watershed data are best considered as an exercise in fitting physical parameters into a theoretical framework. The results were, however, quite encouraging. Figure 10 shows the rainfall pattern for the storm of June 29, 1944, along with the measured and simulated hydrographs. Values for K, C, and the initial saturation profiles were fitted by trial. The roughness parameter used for simulation was $C_1 = 900$, which corresponds to the values for turfed surfaces reported by *Morgali* [1970]. The soil was modeled as a layered system to correspond with the ARS soil sampling data [*Holtan et al.*, 1968].

By comparison with the laboratory flume, runoff rates for the storms simulated on the Hastings, Nebraska, field plots never came near the equilibrium rate for any of the rainfall pulse rates in the storm. On the other hand, runoff from the flume was very close to the rainfall rate minus the infiltration rate by the end of the rainfall pulse. As a result, for simulating the plot watershed response, the surface roughness parameter becomes very important in matching peak rates of runoff.

To test the validity of parameters C_1 and K fitted for this storm, the same valves were applied to another storm on June 5, 1945, initial soil moisture distribution being the remaining fitting parameter. The results for this storm, compared with the recorded hydrograph, are shown in Figure 11.

Each of these storms was described as occur-

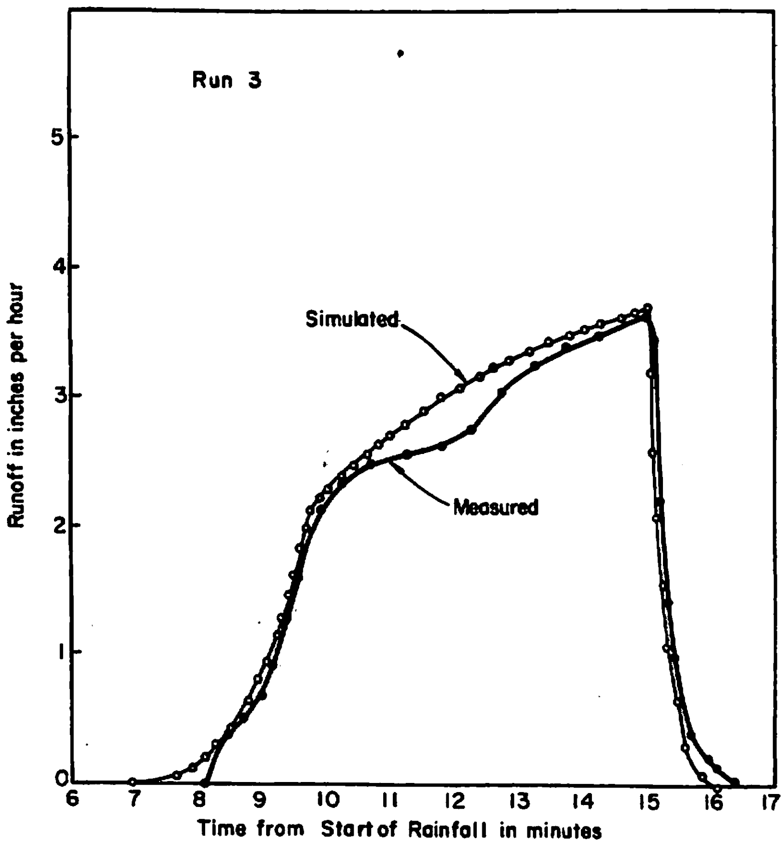

Fig. 8. Measured and simulated hydrographs for run 3 with dry initial conditions.

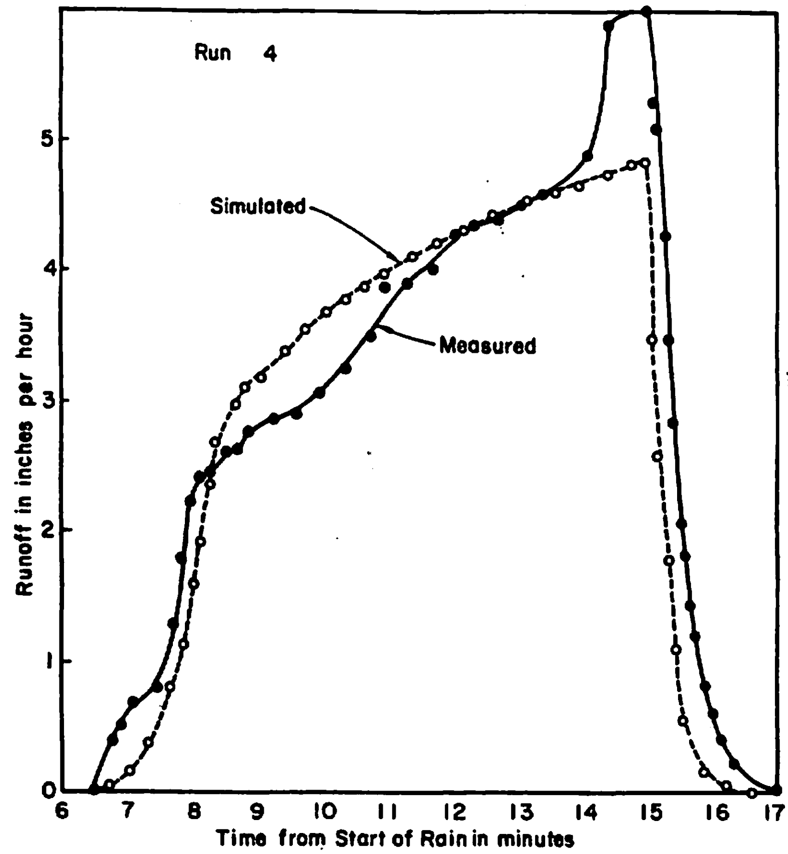

Fig. 9. Measured and simulated hydrographs for run 4 with moist initial conditions.

ring on a 'moist' soil, and indeed the fitted soil saturations used for the results in Figures 10 and 11 are quite similar. Experience with the clocks used in such instrumentation indicates that the coincidence of timing of the rainfall rates and runoff data for the measured hydrographs could be in error by as much as 5 minutes.

The parameters developed for the two storms described above were also applied to a storm for which initial soil saturation data were measured a few days before the storm. This storm (Figure 12) has a double peak, and the soil curves used, which only describe imbibition, should not be able to model well the redistribution of moisture between the two rainfall peaks. The excellent agreement for this storm is in part due to a relatively accurate estimation of initial soil moisture, based on data from 5 days previous. Also it appears from these three storms that recessions are modeled most accurately for storms with little runoff and least accurately for storms with high runoff. This result could be connected with inaccurate estimation of long period infiltration and with the effects of air counterflow. It could also be related to the actual hydraulic effect of the grass as a roughness element or to surface seal development when fine material is being transported in the infiltrating surface water.

Computer running time for these simulations depends on the curvature of the soil moisture-tension curves, the rate of imposed rainfall, and the size of Δz increments. For these simulations the CDC 6400 computer used approximately 1 second for each minute of simulated storm and approximately 70,000$_8$ core storage.

CONCLUSIONS AND RECOMMENDATIONS

The surface runoff response of a watershed to rainfall is a very complex natural process. No theoretical model of a natural watershed can

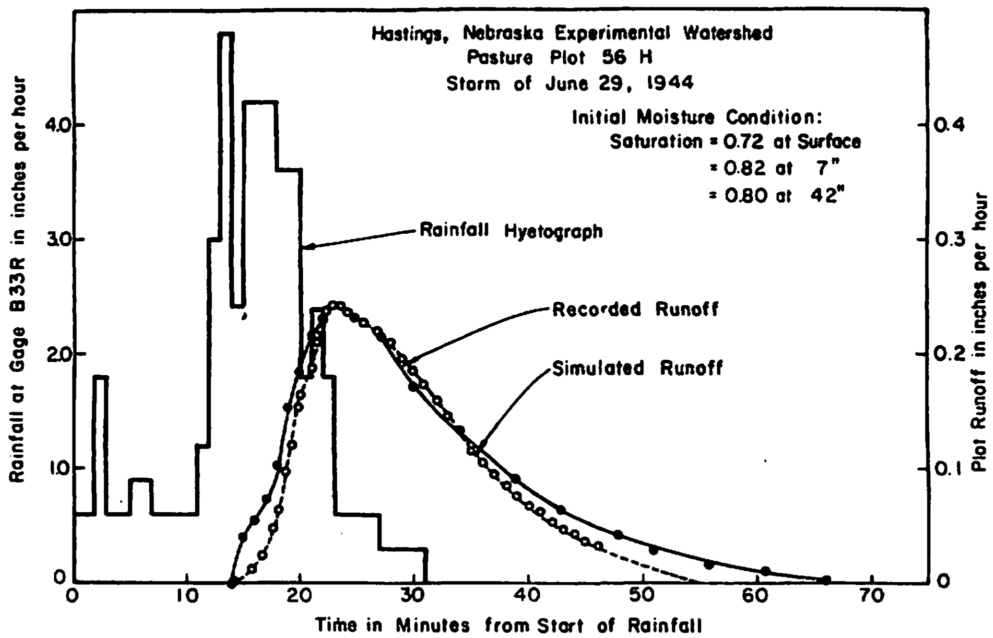

Fig. 10. Rainfall pattern, measured and simulated hydrographs for June 29, 1944, storm on pasture plot 56-H at Hastings, Nebraska.

conceivably be made that could account for all the variables and their interrelationships affecting the runoff process. On the other hand, no model sufficiently simple to be a general engineering tool can hope to model well the result of all these complexities. The objective of model formulation is to make simplifying assumptions so that the model is not unwieldy but still retains the most important characteristics of the physical system.

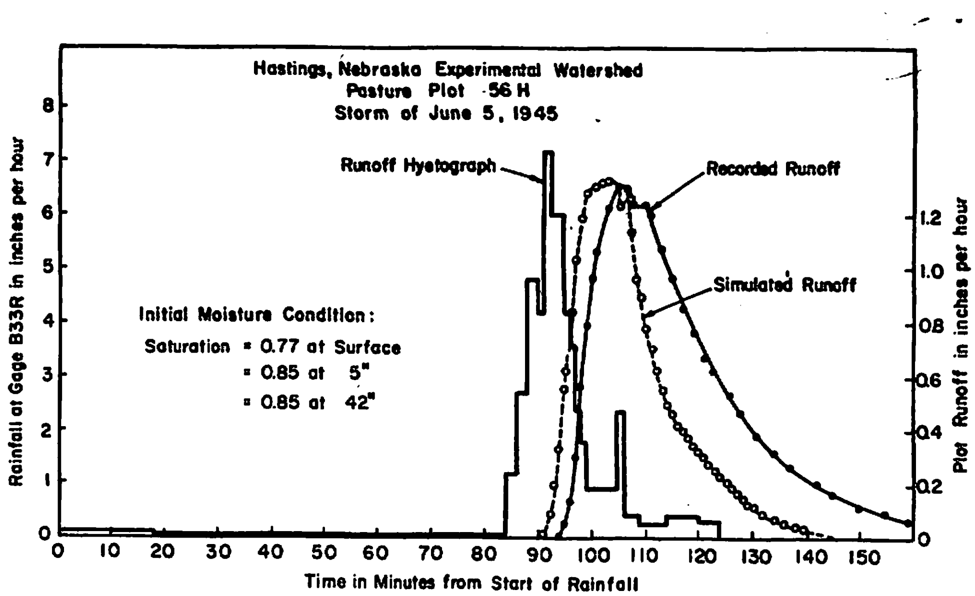

Fig. 11. Rainfall pattern, measured and simulated hydrographs for June 5, 1945, storm on pasture plot 56-H at Hastings, Nebraska.

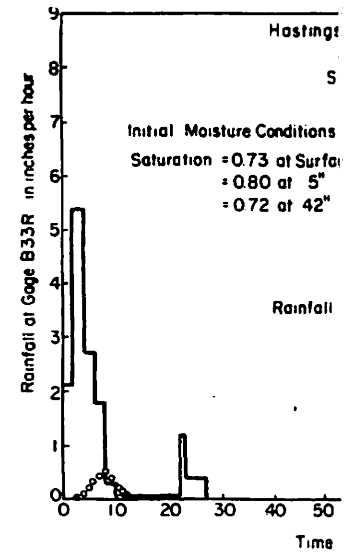

Fig. 12. Rainfall pattern, me
on pas

This study has adopted sufficie
assumptions so that current theorie
movement and watershed hydrau
combined into a mathematical mod
fall–runoff process on small water
The partial differential equation
soil moisture movement was solve
to describe the dependent upper b
ditions governing infiltration. This
demonstrated that this solution ma
with sufficient efficiency and preser
terial continuity to describe the in
as a smoothly varying function o
served in infiltrometer experiment
The combination of this infiltrati
the kinematic equation of overlan
interacting boundary conditions at
face are used, provides a mathemat
the generation of overland flow fr
an infiltrating surface.
If the necessary hydraulic rel
the unsaturated porous media can
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prediction for more complex w
necessarily depend on obtaining re
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soil and watershed surface for de
within the watershed.

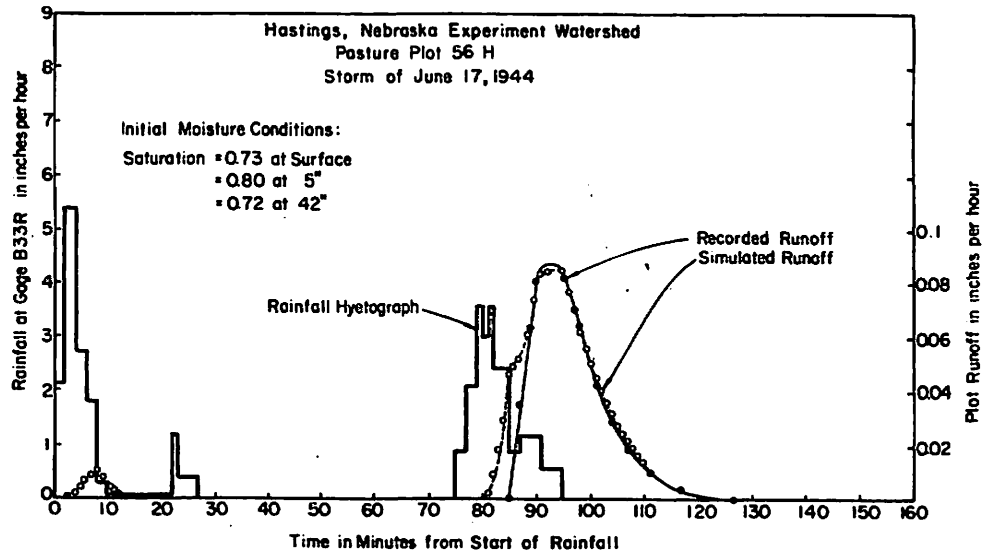

Fig. 12. Rainfall pattern, measured and simulated hydrographs for June 17, 1944, storm on pasture plot 56-H at Hastings, Nebraska.

This study has adopted sufficient simplifying assumptions so that current theories of soil water movement and watershed hydraulics could be combined into a mathematical model of the rainfall–runoff process on small watersheds.

The partial differential equation of one-phase soil moisture movement was solved numerically to describe the dependent upper boundary conditions governing infiltration. This investigation demonstrated that this solution may be obtained with sufficient efficiency and preservation of material continuity to describe the infiltration rate as a smoothly varying function of time as observed in infiltrometer experiments.

The combination of this infiltration model with the kinematic equation of overland flow, when interacting boundary conditions at the soil surface are used, provides a mathematical model for the generation of overland flow from rainfall on an infiltrating surface.

If the necessary hydraulic relationships for the unsaturated porous media can be obtained, this model can accurately describe the performance of a relatively simple infiltrating watershed when rainfall occurs. The accuracy of such a prediction for more complex watersheds will necessarily depend on obtaining reliable data for the effective average hydraulic properties of the soil and watershed surface for definable regions within the watershed.

When soil properties must be estimated, such a model can provide a theoretical framework for a good description of the watershed response, in which system parameters with physical significance, such as effective saturated conductivity and surface roughness, may be obtained by comparison with experimental data.

APPENDIX: FINITE DIFFERENCE EQUATIONS (FROM EQUATION 1)

In Figure 13 let subscripts i refer to finite points in depth, $1 < i < N$, and similarly let superscripts j refer to finite increments of time, $1 < j < \infty$. In the Crank-Nicholson finite difference method, equations are written at time- and space-averaged positions. For a finite difference approximation equation 1 requires six points in the z, t plane (Figure 13). For the averaged points define

$$k_{r_+}^{j-(1/2)} = 1/2(k_{r_i}^{j} + k_{r_i}^{j-1})$$

and

$$k_{r_-}^{j-(1/2)} = 1/2(k_{r_{i-1}}^{j} + k_{r_{i-1}}^{j-1})$$

Providing for variable z increment sizes, let $\Delta\bar{z}_+ = z_{i+1} - z_i$ and $\Delta\bar{z}_- = z_i - z_{i-1}$. From these definitions the implicit finite difference form of equation 1 can be written:

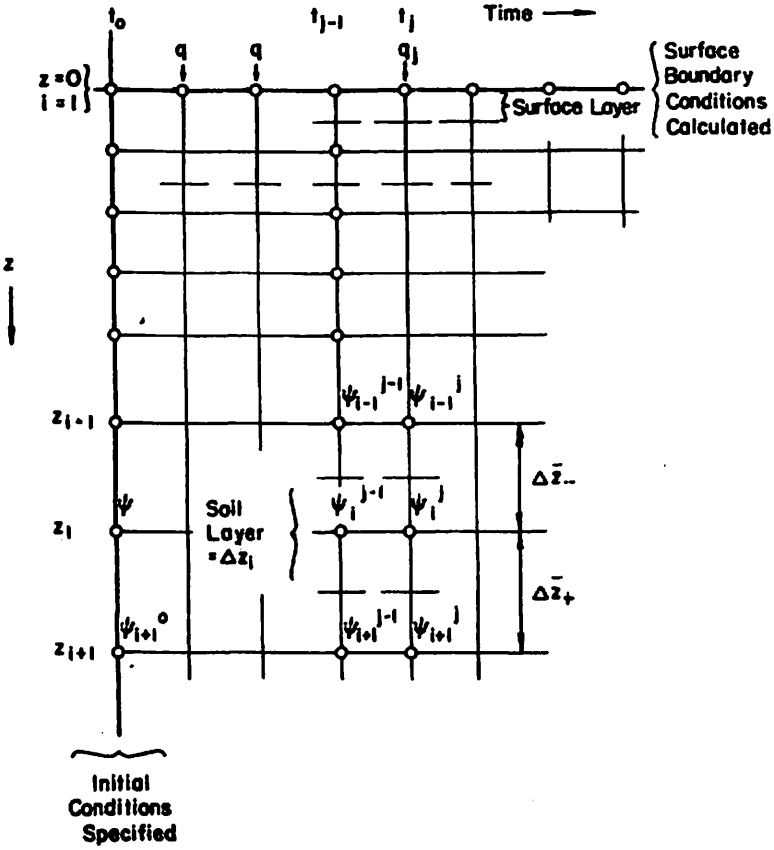

Fig. 13. Finite difference solution grid in (z, t) showing notation used.

$$K_{i+(1/2)}k_{r+}^{\,j-(1/2)} \cdot\left(\frac{\psi_{i+1}^{j} - \psi_i^{j} + \psi_{i+1}^{j-1} - \psi_i^{j-1}}{2\Delta\bar{z}_+}\right) - K_{i-(1/2)}k_{r-}^{\,j-(1/2)} \cdot\left(\frac{\psi_i^{j} - \psi_{i-1}^{j} + \psi_i^{j-1} - \psi_{i-1}^{j-1}}{2\Delta\bar{z}_-}\right) - K_{i+(1/2)}k_{r+} + K_{i-(1/2)}k_{r-} = \left(\frac{\Delta z}{\Delta t}\,\frac{\Delta\phi S_w}{\Delta S}\right)_i^{j-(1/2)}\left(\frac{\Delta S}{\Delta\psi}\right)_i^{j-(1/2)} \cdot(\psi_i^{j} - \psi_i^{j-1}) - QP_i \qquad \text{(A1)}$$

where QP_i is external inflow used for $i = 1$ only.

Terms containing values known from the state of the system at the beginning of the time step $j - 1$ (termed RHS) may be isolated from those to be determined, and the equation may be simplified:

$$\frac{K_{i-(1/2)}k_{r-}}{2\Delta\bar{z}_-}\,\psi_{i-1}^{j} - \left[\frac{K_{i+(1/2)}k_{r+}}{2\Delta\bar{z}_+} + \frac{K_{i-(1/2)}k_{r-}}{2\Delta\bar{z}_-} + \left(\frac{\Delta z}{\Delta t}\,\frac{\Delta\phi S_w}{\Delta S_w}\right)_i^{j-(1/2)}\left(\frac{\Delta S}{\Delta\psi}\right)_i^{j-(1/2)}\right]\psi_i^{j} + \frac{K_{i+(1/2)}k_{r+}}{2\Delta\bar{z}_+}\,\psi_{i+1}^{j} = \mathrm{RHS}_i^{\,j-(1/2)} \qquad \text{(A2)}$$

Combining such expressions for the entire soil column forms a set of simultaneous nonlinear equations. Jacobi iteration is used on the resulting matrix equation,

$$A\psi = \mathrm{RHS} \qquad \text{(A3)}$$

where A is the tridiagonal matrix of coefficients from equation A3, which along with RHS is a function of the solution vector ψ.

REFERENCES

Brooks, R. H., and A. T. Corey, H[...] erties of porous media, *Hydrol.* [...] rado State University, Fort Collin[...]

Burman, Robert D., Plot runoff us[...] wave theory and parameter optim[...] dissertation, Cornell University, [...] York, 1969.

Chow, Ven Te, and T. E. Harbau[...] production for laboratory watersh[...] tation, *J. Geophys. Res., 70*(2[...] 1965.

Foster, G. R., Analysis of overland [...] erosion plots, M.S. thesis, Purd[...] Lafayette, Indiana, January 1968.

Freeze, R. Allan, The mechanis[...] ground-water recharge and disch[...] dimensional, vertical, unsteady, [...] flow above a recharging or disch[...] water flow system, *Water Reso*[...] 153–171, 1969.

Hanks, R. J., and S. A. Bowers, N[...] tion of the moisture flow equati[...] tion into layered soils, *Soil Sc*[...] *Proc., 26*(6), 530–534, 1962.

Holtan, H. N., et al., Moisture-te[...] selected soils on experimental wa[...] *41-144*, U.S. Department of Agr[...] cultural Research Service, Beltsvi[...] October 1968.

Kibler, David F., A kinematic [...] model and its optimization, Ph.[...] Colorado State University, For[...] gust 1968.

Kibler, David F., and David A. [...] kinematic cascade as a hydrologic [...]

REFERENCES

Brooks, R. H., and A. T. Corey, Hydraulic properties of porous media, *Hydrol. Pap. 3*, Colorado State University, Fort Collins, March 1964.

Burman, Robert D., Plot runoff using kinematic wave theory and parameter optimization, Ph.D. dissertation, Cornell University, Ithaca, New York, 1969.

Chow, Ven Te, and T. E. Harbaugh, Raindrop production for laboratory watershed experimentation, *J. Geophys. Res.*, *70*(24), 6111–6119, 1965.

Foster, G. R., Analysis of overland flow on short erosion plots, M.S. thesis, Purdue University, Lafayette, Indiana, January 1968.

Freeze, R. Allan, The mechanism of natural ground-water recharge and discharge, 1, One-dimensional, vertical, unsteady, unsaturated flow above a recharging or discharging ground-water flow system, *Water Resour. Res.*, *5*(1), 153–171, 1969.

Hanks, R. J., and S. A. Bowers, Numerical solution of the moisture flow equation for infiltration into layered soils, *Soil Sci. Soc. Amer. Proc.*, *26*(6), 530–534, 1962.

Holtan, H. N., et al., Moisture-tension data for selected soils on experimental watersheds, *ARS 41-144*, U.S. Department of Agriculture, Agricultural Research Service, Beltsville, Maryland, October 1968.

Kibler, David F., A kinematic overland flow model and its optimization, Ph.D. dissertation, Colorado State University, Fort Collins, August 1968.

Kibler, David F., and David A. Woolhiser, The kinematic cascade as a hydrologic model, *Hydrol. Pap. 39*, Colorado State University, Fort Collins, March 1970.

Klute, Arnold, A numerical method for solving the flow equation for water in unsaturated materials, *Soil Sci.*, *73*(2), 105–116, 1952.

Morgali, James R., Laminar and turbulent overland flow hydrographs, *J. Hydraul. Div., Amer. Soc. Civil Eng.*, *96*(HY2), *Proc. Pap. 7089*, February 1970.

Philip, J. R., The theory of infiltration, 1, The infiltration equation and its solution, *Soil Sci.*, *83*(5), 105–116, 1957.

Rubin, J., Theory of rainfall uptake by soils initially drier than their field capacity and its application, *Water Resour. Res.*, *2*(4), 739–749, 1966.

Rubin, J., and R. Steinhardt, Soil–water relations during rain infiltration, 1, Theory, *Soil Sci. Soc. Amer. Proc.*, *27*(3), 246–251, 1963.

Smith, R. E., Mathematical simulation of infiltrating watersheds, Ph.D. dissertation, Colorado State University, Fort Collins, June 1970.

Whisler, F. D., and A. Klute, The numerical analysis of infiltration, considering hysteresis, into a vertical soil column at equilibrium under gravity, *Soil Sci. Soc. Amer. Proc.*, *29*(5), 489–494, 1965.

Wooding, R. A., A hydraulic model for the catchment-stream problem, 1, Kinematic-wave theory, *J. Hydrol.*, *3*(3–4), 254–267, 1965.

Woolhiser, D. A., and J. A. Liggett, Unsteady, one-dimensional flow over a plane—The rising hydrograph, *Water Resour. Res.*, *3*(3), 753–771, 1967.

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