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# A Note on the Muskingum Flood-Routing Method

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*Abstract*—An exact method of solution of the flood-routing equation, when the storage is a linear function of weighted inflow and outflow, is developed. This operation is shown to be equivalent to routing a multiple of the inflow through reservoir storage and subtracting the excess inflow. Modified coefficients for the Muskingum equation are developed which do not depend on the routing interval being small relative to $K$.

*Introduction*—The Muskingum method, which is a finite difference method of solution of the flood routing equation, under the assumption that storage is a linear function of a weighted mean of inflow and outflow, $S = K[xI + (1 - x)Q]$, is widely used, both in its original form [*McCarthy*, 1938] and as the basis of a number of graphical or semi-graphical methods. In the use of these methods, however, it is sometimes overlooked that an essential requirement, to insure accuracy in such finite difference calculations, is that the finite interval $T$ must be small relative to the other time elements involved. This fact was emphasized by *Clark* [1945] in discussing the Muskingum flood-routing method. Nevertheless, it still happens that values of $T$ sensibly equal to $K$ are recommended for use in actual calculation.

Whereas it is possible that in practice the inaccuracies so introduced are generally not significant relative to the inaccuracies introduced by the basic storage assumption, and the usual inaccuracies of the data, it may happen, particularly in theoretical work, that a high relative accuracy is required. The failure of the Muskingum method when $T/K$ is not small is demonstrated by the widely accepted belief that routing through linear storage with $x = 0.5$ operates as a pure delay. This conclusion is based on the fact that the substitution of $T = K$ and $x = 0.5$ in the Muskingum equation yields $Q_1 = I_0$, the other coefficients being zero. That this conclusion is erroneous is demonstrated in this note. An exact method of solution under the storage assumption is developed, and modified equations for the Muskingum coefficients are derived. These equations are true even when $T$ is not small relative to $K$.

*Notation*—

$I(t)$ = inflow, ft³/sec
$Q(t)$ = outflow, ft³/sec
$x$ = a numerical parameter
$S(t)$ = storage, ft³ hrs/sec
$K$ = a time parameter, hours
$k$ = $K(1 - x)$
$D$ = the differential operator $d/dt$
$C_1$ $C_2$ $C_3$ the Muskingum coefficients
$c$ = $\exp - T/K(1 - x)$
$q$ = the outflow from routing $I$ through $S = K(1 - x)q$
$T$ = the routing interval, hours
$m$ = slope of inflow curve

*The exact solution*—The fundamental equations are

$$I = Q + \frac{ds}{dt} \qquad (1)$$

$$S = K(xI + (1 - x)Q) \qquad (2)$$

from which

$$I - xK\frac{dI}{dt} = Q + (1 - x)K\frac{dQ}{dt}$$

$$Q(t) = \frac{1 - xKD}{1 + (1 - x)KD} I(t) \qquad (3)$$

When $x$ = zero we have the corresponding reservoir case

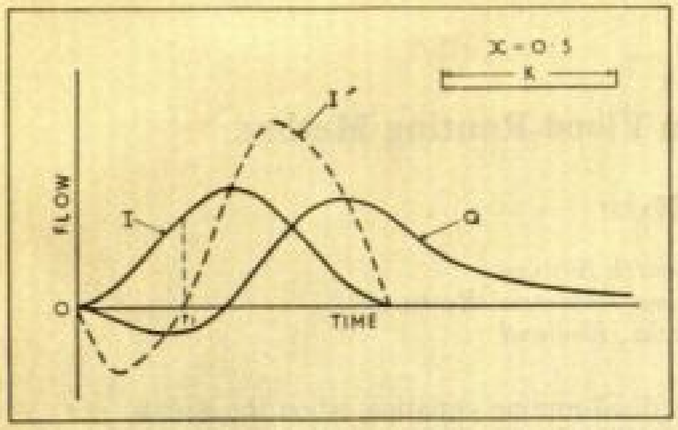


FIG. 1—Routing through storage with $x = 0.5$

$$Q(t) = \frac{1}{1 + KD} I(t) \quad (4)$$

which has the solution

$$Q = \frac{1}{K} e^{-t/K} \int e^{t/K} I \, dt \quad (5)$$

Now (3) may be looked upon as the result of operating on $I(t)$ successively with $1 - xKD$ and $1/[1 + (1 - x)KD]$. The operation $1 - xKD$ merely involves differentiation of the inflow and $1/[1 + (1 - x)KD]$ represents reservoir routing with $S = (1 - x)KQ$. Therefore (3) is equivalent to subtracting $xK$ times the first derivative of $I$ from $I$ and routing the remainder through reservoir storage with $S = (1 - x)KQ$. These operations generally can be carried out graphically, and mathematically as well, when $I$ is a simple function of time. We can learn more from (3). Let us define

$$I'(t) = (1 - xKD)I(t) \quad (6)$$

This means that $I'$ is the result of routing $I$ backwards (that is, calculation of inflow from outflow) through linear reservoir storage $S = -xKI$. The effect of the negative $xK$ is achieved by taking the routing procedure from right to left; that is, in the negative direction of time (Fig. 1).

When we come to $t_1$ at which $I'$ becomes zero, $I$ would fall off logarithmically and never actually reach zero unless $I'$ took negative values. This means that when $I$ starts from zero and rises at a finite rate, $I'$ must always take negative values initially.

It is clear, too, that the interval between the centers of area of $I'(t)$ and $I(t)$ is $xK$. We must now route $I'$ forwards through $S = (1 - x)KQ$ to obtain $Q$ (Fig. 1). Clearly this involves a further shift of the center of area $(1 - x)K$ so that the total shift is $K$. However $I$ and $Q$ are not otherwise identical even when $x = 0.5$ as shown in Figure 1. It should be noted that the negative initial values of $I'$ result in negative initial values of $Q$.

If we divide out the operator in (3) we obtain

$$Q = \left[ -\frac{x}{1 - x} + \frac{1}{(1 - x)\{1 + (1 - x)KD\}} \right] I \quad (7)$$

$$Q = \frac{1}{1 + (1 - x)KD} \frac{I}{(1 - x)} - \frac{xI}{1 - x} \quad (8)$$

We see, therefore, that the outflow consists of the sum of two parts, the first of which we shall call $q$, being the result of routing $I/(1 - x)$ through $S = K(1 - x)q$, and the second part being simply the inflow multiplied by $-x/(1 - x)$.

There are various ways of routing through reservoir storage. Equation (5) may always be integrated graphically, or mathematically if $I$ is in a suitable form. A simple graphical solution not involving integration has been demonstrated by *Nash* and *Farrell* [1955]. It frequently happens, however, that a coefficient solution is desired. Formulas for the coefficients are calculated in the next section.

*Modification of the Muskingum coefficients*—

$$Q_1 = C_1 I_0 + C_2 I_1 + C_3 Q_0 \quad (9)$$

We shall use (8) to obtain the expression for the $C$'s. In expressing $Q$ as a function of $I_0$ $I_1$ and $Q_0$ only, we must neglect second and higher derivatives of $I$; that is, we must assume $I$ to consist of straight-line segments. If the second or higher derivatives are required, we must use three or more values of $I$ in (9). However, by choosing time intervals which are sufficiently short, the calculation using only $I_0$, $I_1$, and $Q_0$ can be made as precise as is desired. The only difference between the present calculation and the usual development of the Muskingum coefficient equation is that we are not limited to values of the time interval which are small compared with $K$.

The solution of (8) when $I$ is a series of straight segments is obtained as follows. Let $m = (I_1 - I_0)/T$ be the slope of a segment.

Let

$$q(t) = \frac{1}{1 + (1 - x)KD} I(t)$$

then

$$Q = q/(1 - x) - xI/(1 - x) \qquad (10)$$

Let $k = (1 - x)K$ and $c = \exp[-T/K(1 - x)]$ to simplify the notation. From (5)

$$q = 1/ke^{-t/k} \int (I_0 + mt)e^{t/k}\, dt$$

$$q = 1/ke^{-t/k}[kI_0e^{t/k} + mk^2e^{t/k}(t/k - 1) + A]$$

$$q = I_0 + mk(t/k - 1) + A/ke^{-t/k} \qquad (11)$$

We solve for the arbitrary constant $A$ by letting $q = q_0$ at $t = 0$ and obtain $q_0 = I_0 - mk + A/K$. Substituting in (11) we obtain

$$q = I_0 + mk(t/k - 1) + (q_0 - I_0 + mk)e^{-t/k}$$

Substituting $(I_1 - I_0)/T$ for $m$ and letting $t = T$, we obtain

$$q_1 = I_0 + k/T(T/k - 1)(I_1 - I_0) + [q_0 - I_0 + k/T(I_1 - I_0)]c$$

$$q_1 = I_0[k/T(1 - c) - c] + I_1[-k/T(1 - c) + 1] + q_0c \qquad (12)$$

whence by (10)

$$Q_1 = I_0\left[\frac{k}{T}\frac{1 - c}{1 - x} - \frac{c}{1 - x}\right] + I_1\left[-\frac{k}{T}\frac{1 - c}{1 - x} + \frac{1}{1 - x} - \frac{x}{1 - x}\right] + q_0\frac{c}{1 - x}$$

But $q_0/(1 - x) = Q_0 - xI_0/(1 - x)$ which when substituted in (12), bearing in mind that $k = K(1 - x)$, gives

$$Q_1 = I_0[K/T(1 - c) - c] + I_1[-K/T(1 - c) + 1] + Q_0c \qquad (13)$$

This is the modified form of the Muskingum equation when $T$ is not small relative to $K$. If $T/K$ is taken very small the coefficients in (13) and in the Muskingum equation converge.

*Conclusions*—We have seen that the Muskingum method is equivalent to either routing the inflow backwards (that is, calculating $I$ from $Q$) through storage $S = -xKI$ and subsequently forwards through $S = (1 - x)KQ$, or routing a multiple of the inflow $I/(1 - x)$ through $S = K(1 - x)q$ and subtracting $xI/(1 - x)$. We have also developed the values of the coefficients to be used when $T/K$ is not small. The negative outflow, which may occur when the inflow rises steeply, has been explained as being essentially associated with the storage assumption, and not with any particular method of solution. This rather unrealistic consequence of the storage assumption suggests that some modification is desirable. As it is necessary in practice to determine $x$ and $K$ by experiment, it would seem more reasonable to abandon the storage assumption and consider the linear operation to consist of two parts, a pure delay and a single reservoir routing [*Hopkins*, 1956], the two parameters to be determined by experiment. The pure delay plus the storage factor $K$ would be equal to the lag between the centers of area of inflow and outflow, and the ratio of the storage factor to the lag would form a dimensionless parameter which might be constant as a first approximation or reflect some characteristics, at present unknown, of the channel. It might be possible to determine this relation by means of a statistical correlation.

*Acknowledgment*—This note is published by permission of the Director, Hydraulics Research Station, Department of Scientific and Industrial Research, Howbery Park, Wallingford, Berkshire, England.

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# Systematic Determination of Unit Hydrograph Parameters

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*Abstract*—In unit hydrograph correlations the number of degrees of freedom which it is useful to maintain in the form of the instantaneous unit hydrograph is shown to be limited by the number of significant independent correlations with the catchment characteristics. The moments of the instantaneous unit hydrograph are suggested as a series of parameters of the response, for which correlations should be sought. A simple method of obtaining these moments is evolved, and a method of choosing, in any given case, between several two-parameter forms is demonstrated.

*Notation*—The following notation is used:

IUH—instantaneous unit hydrograph
$i$—input (effective rainfall) a function of time
$u$—the indicial response (instantaneous unit hydrograph) a function of time
$T$—duration of inflow
$V$—storage
$K$—a linear storage parameter
$s$—output (storm runoff), a function of time
$a$, $b$, $c$—times from the origin to the centers of area of the input, the output, and the indicial response respectively
$\tau$, $\epsilon$, $\gamma$—time measured from the centers of area of input, indicial response, and output, respectively
$I_n$, $U_n$, $S_n$—the $n$th moments about the centers of area of the input, indicial response, and output, respectively
$I_n'$, $U_n'$, $S_n'$—corresponding moments about the origin
$m_n = U_n/c^n$—the $n$th dimensionless moment of the indicial response
$h$, $g$—parameters in the equation of the log-normal frequency distribution

*Number of parameters*—The essence of the unit hydrograph theory is the assumption of linearity in the relation between storm runoff and effective rainfall. Having made this assumption, it remains to establish the relation between the characteristics of the catchment and the response in the storm runoff to a predetermined 'input' of effective rainfall, that is, the indicial response. The most frequently used indicial response is the IUH, defined as the hydrograph of storm runoff caused by unit volume of effective rainfall, generated instantaneously and uniformly, over the catchment. To establish the relationship between the IUH and the catchment characteristics by means of a statistical correlation, it is necessary to define the IUH and the catchment characteristics by numerical parameters. The parameters of the IUH may be actual measurements of, or constants in, the assumed equation of the IUH. Correlations must be established between the IUH parameters as dependent variables and the catchment characteristics as independent variables.

Such correlations may be classified as 'one parameter,' 'two parameter,' . . . depending on the number of parameters of the IUH for which independent correlations are obtained. Obviously, the greater the number of parameters we succeed in correlating the better, as we are thus enabled to use a more general IUH form.

If we measure only a single parameter, then we must assume that all IUH's having equal values of this parameter are identical. If we use two parameters, we can obtain a better fit to an actual IUH, due to the greater generality of the two-parameter form. It is not nesessary, though it is desirable, that the form of IUH be expressed as an actual algebraic equation $u = u(t)$ (discharge as a function of time). For example, one could measure the peak and assume that the shape is given by *Commons* [1942] basic hydrograph (one parameter), or one could assume that the IUH was generated by routing an isosceles triangular inflow of base $T$ hours through linear reservoir storage, $V = Ks$ (two parameters).

Clearly, the greater generality of a many-parameter form can be availed of, only if significant independent correlations are obtained for each parameter. In the two-parameter case mentioned (namely, where the IUH is assumed to be generated by routing an isosceles triangular inflow of base $T$ hours through linear reservoir storage), if correlations for $K$ and $T$ are obtained with different combinations of catchment characteristics we have a genuine two-parameter correlation. However if both $T$ and $K$ are correlated with the same catchment characteristics in such a manner that the catchment characteristics can be eliminated between the two equations and $K$ expressed as a function of $T$, then we have only a one-parameter correlation because $K$ can be replaced by the corresponding function of $T$.

Intermediate between these two cases we might find $K$ and $T$ correlated with almost the same catchment characteristics. In such a case it might be difficult to say whether we had achieved a one- or two-parameter correlation.

There are two precautions which help to prevent this situation arising. We may take as our two parameters $T$ and $T/K$ which are much less likely to be mutually correlated than $T$ and $K$. We may also include the first parameter among the independent variables when studying the regression of the second parameter on the catchment characteristics. If a multiple linear regression is used, the values of the $\beta$ coefficients [*Ezekiel*, 1941] show the relative proportions of the variance of the second dependent variable which is attributable to variations in the first parameter and to variations in the catchment characteristics themselves.

*Moments as parameters*—In defining the IUH parameters to use in the correlation we must remember that the parameters must be evaluated, by some curve-fitting process, from the records for each catchment used in the correlation. Therefore the parameters must be stable, in that small errors in the records must not produce large variations in the parameters. $T$ and $K$ of the routed isosceles triangle are not stable in this sense. Obviously a fairly large change in $T$ and $K$ in opposite directions would produce only a small change in the IUH generated, and conversely. In fact if we wished to use this particular two-parameter form we should correlate $(T + K)$ and $T/K$ with the catchment characteristics. $(T + K)$ could be obtained from the records with greater certainty than either $T$ or $K$. The second point to be remembered is the ease or difficulty with which the parameters may be evaluated from the records. For example, if we use the peak of the IUH as a parameter, it is necessary to derive from the records the actual IUH for each catchment, a very difficult task.

It is the author's suggestion that the form of the IUH should not be determined until after the correlation, and that the moments of the IUH, which can be obtained very easily from the records without deriving the actual IUH, should be used as the parameters of the response. It is shown in Appendix 1 that the first moment of the IUH about the instant of effective rainfall is equal to the difference between the first moments of the storm runoff and effective rainfall about the time of beginning of effective rainfall. It is also shown that the second and third moments of the IUH about its center of area, are respectively equal to the differences between the corresponding moments of storm runoff and effective rainfall, each about its own center of area. The corresponding relation for higher moments is also derived.

Thus, to obtain as many parameters of the IUH as we wish, we simply calculate the moments of the storm runoff and effective rainfall and apply (1) and (4) of Appendix 1.

To obtain independence between the parameters of the IUH it is better to use, instead of the $n$th moment, the ratio of the $n$th moment to the first moment in the power of $n$. This renders all the parameters except the first dimensionless. In the notation defined above, the parameters become $c, m_2, m_3, \ldots$. These parameters should be derived for each catchment and treated as dependent variables. Having obtained a correlation between $c$ and the catchment characteristics, a second correlation should be sought between $m_2$ as dependent variable, and $c$ and the catchment characteristics as independent variables, and so on.

At some value of $n$ it will be found that $m_{n+1}$ shows no significant correlation with the catchment characteristics. It is possible that it might show significant correlation with $c, m_2, m_3, \ldots m_n$, but this is unimportant at this stage. We would thus have achieved an $n$ parameter correlation and we must then seek an $n$ parameter form for the IUH. To decide between various forms we might use the chi-squared test, used by statisticians to measure the closeness of fit of a frequency distribution. If $n$ is small, as unfortunately is most probable, a method similar to that shown in Appendix 2 might be used to judge the suitability of various two-parameter forms. Here the ability of each form to predict the third moment, when its two parameters are determined by equating first and second moments, is used as a test of suitability.

Concerning the number of parameters for which we might hope to obtain correlations it may be noted that the correlations of *Bernard* [1935], *McCarthy* [1940], *Snyder* [1938], and *O'Kelly* [1955] are all 'one parameter' while that of *Taylor* and *Schwartz* [1953] is 'two parameter.'

## Appendix 1—The Effect of a Linear Transformation on Moments

The object here is to establish the relationship between the moments of the 'input,' the 'indicial response' to a unit pulse, and the 'output.' Let us assume that the input and output are expressed in units which make $\int i\,d\tau$ and $\int s\,d\gamma$ both unity. This does not limit the generality of the equations subsequently derived, but it simplifies the expressions slightly.

In the transformation, the elementary strip $i\,d\tau$ (Fig. 1), is replaced by an elementary output whose center of area must be later than $\tau$ by $c$. This applies to every elementary strip; therefore the center of area of the output must be later than that of the input by $c$.

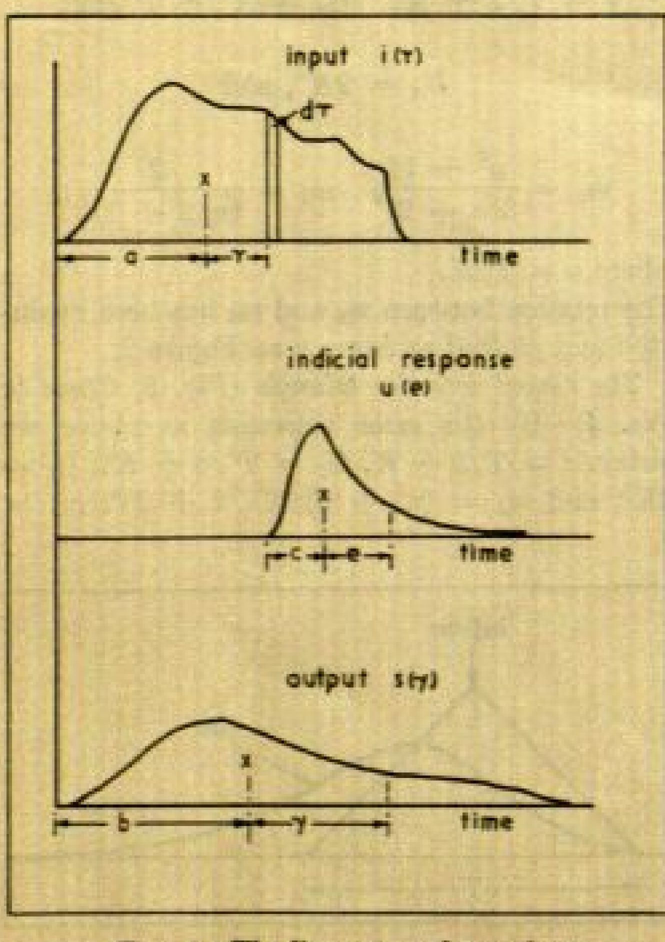


Fig. 1—The linear transformation

$$b = a + c \qquad (1)$$

This is the relation between the first moments about the origin. The relation between the $n$th moments about the centers of area is found as follows.

Du Hamel's integral gives the output at time $\gamma$ as

$$s = \int i(\tau)u(\epsilon)\,d\tau \quad \text{where} \quad \gamma = \epsilon + \tau.$$

$$\therefore\ S_n = \iint i(\tau)u(\epsilon)(\epsilon + \tau)^n\,d\tau\,d\epsilon \qquad (2)$$

which may be expanded as

$$S_n = \iint i(\tau)u(\epsilon)\Big(\epsilon^n + n\epsilon^{n-1}\tau + \frac{n(n-1)}{2!}\epsilon^{n-2}\tau^2 + \cdots + \tau^n\Big)\,d\epsilon\,d\tau.$$

Now

$$U_n = \int u(\epsilon)\epsilon^n\,d\epsilon, \quad \text{and} \quad I_n = \int i(\tau)\tau^n\,d\tau.$$

$$\therefore\ S_n = \int i(\tau)\Big(U_n + nU_{n-1}\tau + \frac{n(n-1)}{2!}U_{n-2}\tau^2 + \cdots \tau^n\Big)\,d\tau.$$

$$\therefore\ S_n = U_n + nU_{n-1}I_1 + \frac{n(n-1)}{2!}U_{n-2}I_2 + \cdots I_n. \qquad (3)$$

If we write the suffixes as power indices (without of course interpreting them as such, except for the purpose of expansion), we get

$$S_n = (U + I)^n. \qquad (4)$$

This equation is quite general. It has very simple expansions for small values of $n$. Remembering that $U_1 = I_1 = S_1 = 0$ we get

$$S_2 = I_2 + U_2 \qquad (5a)$$

$$S_3 = I_3 + U_3 \qquad (5b)$$

$$S_4 = I_4 + U_4 + 6I_2U_2 \qquad (5c)$$

Eq. (4) and (1) enable us to calculate the moments of the IUH very simply from the moments of the storm runoff and effective rainfall.

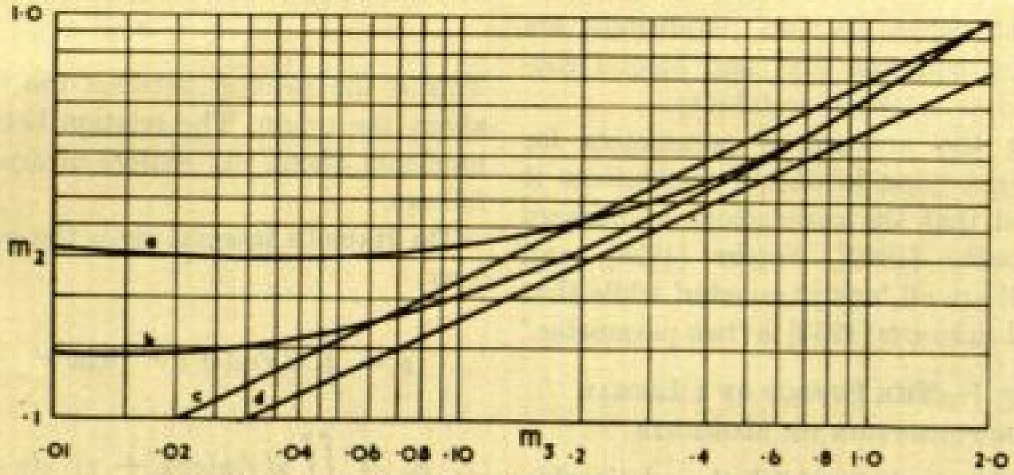


FIG. 2—Comparison of certain two-parameter forms

### APPENDIX 2—CALCULATION AND COMPARISON OF THE MOMENTS OF CERTAIN TWO-PARAMETER FORMS

Here the quantities $c$, $m_2$, and $m_3$ are calculated for a number of possible two-parameter IUH forms. The relationship between $m_3$ and $m_2$ is calculated and plotted as a set of curves (Fig. 2). If we obtain, as described earlier in this paper, the values of $m_2$ and $m_3$ for each of several catchments, we can decide which of the two-parameter forms gives the best representation of the several IUH's by plotting corresponding values of $m_3$ and $m_2$ on Figure 2. The form corresponding to the line which passes closest to the scatter so obtained is the most suitable form.

*The routed rectangle (Fig. 2, Curve a)*—Figure 3

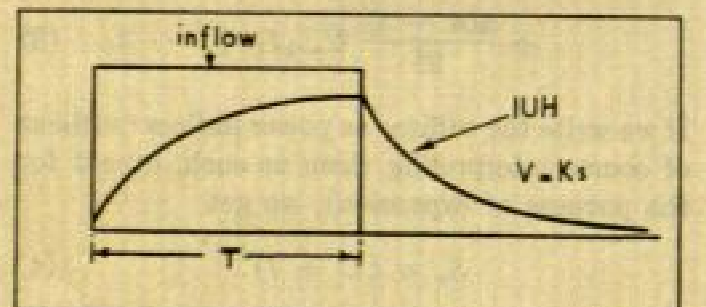


FIG. 3—The routed rectangle

shows the method of generation of this form. A rectangular inflow of duration $T$ is routed through linear storage $V = Ks$.

The process of routing through linear storage is a linear transformation, and therefore the equations developed in Appendix 1 may be used to obtain the moments of the routed rectangle. In this case the response to a 'unit pulse' input is given by $u = e^{-t/K}/K$

$$U_n' = \int_0^\infty \frac{t^n}{K} e^{-t/K}\, dt$$

$$= K^n \int_0^\infty \left(\frac{t}{K}\right)^n e^{-t/k}\, d(t/K) = K^n\Gamma(n)$$

whence $c = K$, $U_2 = K^2$, and $U_3 = 2K^3$. The moments of the input are $a = T/2$, $I_2 = T_2/12$, and $I_3 = 0$ whence by (1) and (5), $b = T/2 + K$, $S_2 = K^2 + T^2/12$, and $S_3 = 2K^3$. This gives us the moments of the IUH of curve $a$ of Figure 2 as

$$c = T/2 + K, \quad U_2 = K^2 + T^2/12,$$

$$U_3 = 2K^3, \text{ and}$$

$$m_2 = \frac{n^2 + 12}{3(n + 2)^2}, \quad m_3 = \frac{2}{(n/2 + 1)^3},$$

where $n = T/K$.

The relation between $m_2$ and $m_3$ has been evaluated and plotted as curve $a$ on Figure 2.

*The routed isosceles triangle (Fig. 2, Curve b; Fig. 4)*—By the same reasoning as above we obtain $c = T/2 + K$, $U_2 = T^2/6 + K^2$, $U_3 = 2K^3$, and $m_2 = (n^2 + 6)/6(n/2 + 1)^2$, $m_3 =$

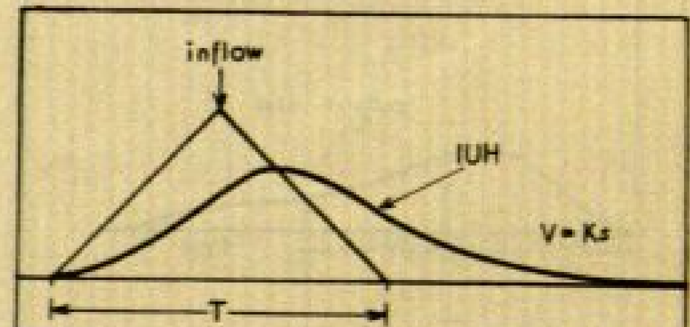


FIG. 4—The routed isosceles triangle

$2/(n/2+1)^3$. The relation between $m_2$ and $m_3$ is shown as curve b on Figure 2.

*Successive routings*—If an instantaneous inflow is routed through a series of $n$ linear reservoirs, each characterized by $V = Ks$, the outflow takes the form

$$u = \frac{V}{K\Gamma(n)} e^{-t/k}(t/K)^{n-1}.$$

The author has suggested elsewhere [*Nash*, 1958], the possibility of using this equation for the IUH. By repeated application of (1) and (5) we get $c = nK$, $U_2 = nK^2$, $U_3 = 2nK^3$, $m_2 = 1/n$, $m_3 = 2/n_2 = 2m_2^2$. The relation between $m_2$ and $m_3$ is shown as curve c on Figure 2.

*The equation of the log-normal frequency distribution*—The author has noticed that $S$ curves (the integral of the IUH) are frequently nearly straight lines when plotted on log-normal probability paper. This implies an equation for the IUH of form

$$u = \frac{1}{t\sqrt{h\pi}} e^{\frac{-(\overline{\log t} - g)^2}{h}}$$

where $g$ and $h$ are parameters. *Chow* [1955] gives the moments of this curve about the origin as

$$U'_n = \exp\left(\frac{hn^2}{4} + ng\right),$$

from which it can be shown that

$$c = e^{h/4} + g,$$

$$U_2 = e^{h+2g} - e^{h/2+2g},$$

and

$$U_3 = e^{9h/4+3g} - 3e^{h/4+g}(e^{h+2g} - e^{h/2+2g}) - e^{3h/4+3g}.$$

Whence $m_2 = e^{h/2} - 1$, $m_3 = e^{3h/2} - 3e^{h/2} + 2 = m_2^3 + 3m_2^2$. The relation between $m_2$ and $m_3$ is shown as curve d on Figure 2.

For the routed rectangle and triangle there are two possible values of $m_3$ for each value of $m_2$. This means that in fitting these forms to an actual IUH by equating first and second moments there are two possible values of $T/K$ each of which gives exactly the same second moments. This is not really a disadvantage, as the position of the point corresponding to the actual IUH on Figure 2 shows which value of $m_3$ (and therefore of $T/K$) gives the best fit.

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